Rate-maximization Channel Assignment Scheme in Cognitive Radio Networks

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Abstract. In this paper, we consider the coordinated spectrum access problem in a multiuser single-transceiver cognitive radio network (CRN). Our objective is to maximize the sum-rate achieved by all contending CR users with respect to both spectrum assignment and transmission rate. The problem is posed as a utility maximization problem subject to hardware and interference constraints. Specifically, we show that this problem can be formulated as an integer linear programming problem (ILP) with unimodular constraint matrix, which can be optimally solved in polynomial time using linear programming (LP). Unlike previous research, our formulation is not restricted to the information-theoretic capacity, and can be applied to any arbitrarily given rate-SINR function. We also develop a distributed CSMA/CA-based MAC protocol for CRNs to realize the optimal assignment in a distributed manner. The performance of our protocol is compared to a reference CSMA/CA CRN MAC protocol. Simulation results show that the proposed protocol significantly improves network throughput and preserves fairness.

1 Introduction

Spectrum measurements by the FCC Spectrum Policy Task Force [1] indicate significant temporal and geographical variations in the utilization of the licensed spectrum, ranging from 15% to 85% [2]. To effectively utilize highly underutilized portions of the spectrum (a.k.a, spectrum holes, etc.), the FCC is revising its static spectrum allocation policies to allow for opportunistic on-demand spectrum access. Toward this end, cognitive radios (CRs) have been proposed as a key technology to allow for such opportunistic on-demand access to the limited spectrum resources without negatively affecting the spectrum-licensed primary-radio (PR) users. CRs are smart, programmable radios that are capable of channel-interference sensing, environment learning, and dynamic spectrum access [2].
A CR network (CRN) should dynamically utilize spectrum opportunities (idle PR channels) and adapt its operating parameters according to the surrounding environment such that the PRN’s performance is not negatively affected. Specifically, CR users should frequently sense their operating frequency channels for active PR users, and should vacate these channels if a PR signal is detected. Thus, to enable an efficient OSA, the operation of a CRN should address two essential issues: (1) identifying spectrum opportunities (i.e., idle PR channels that are potentially available for CR transmissions), and (2) providing an efficient reuse of such opportunities (achieving the maximum possible CRN throughput). While several studies have focused on the first issue based on channel sensing in a cooperative or non-cooperative manner, the second issue is still a challenging problem in a multi-user CRN. Thus, in this paper, we tackle the following question: “given the available spectrum opportunities at different CR users, what is the optimal channel assignment strategy in a multi-user single-transceiver CRN that efficiently utilizes those opportunities and achieves the maximum possible network throughput”?

1.1 Related Work

Recently, several attempts were made to develop channel assignment/access mechanisms for CRNs (e.g., [3–13]). The spectrum access mechanism in [4] jointly optimizes the multi-channel power/rate assignment, assuming a given power mask on CR transmissions. DDMAC [5] is a spectrum-sharing protocol for CRNs that attempts to maximize the number of simultaneous CR transmissions through a novel probabilistic channel assignment algorithm that exploits the dependence between the signal’s attenuation model and the transmission distance while considering the prevailing traffic and interference conditions. DDMAC assumes a given rate demand per a CR user. AS-MAC [6] is a spectrum-sharing protocol for CRNs that coexist with a GSM network. CR users select channels based on the CRN’s control exchanges and GSM broadcast information. Explicit coordination with the PRNs is required. In [12], the authors developed a spectrum aware channel access protocol for CRNs (CMAC). CMAC enables opportunistic access and sharing of the available white spaces in the TV spectrum by adaptively allocating the spectrum among contending users. In [14], the interference temperature model [15] is used to select an optimal bandwidth/power assignment for CR users. COMAC [3] is a distributed MAC protocol for opportunistic CRNs. It improves spectrum utilization while statistically guaranteeing the performance of PR users. Specifically, COMAC ensures a statistical (soft) guarantee on the performance of PRNs. In [16], the authors developed a power control approach for CR systems based on spectrum sensing side information. The objective of such an approach is to mitigate the interference to a PR user from CR transmissions. Three spectrum sharing techniques were
proposed and compared in [17]: spreading-based underlay, interference avoidance overlay, and spreading-based underlay with interference avoidance. The metric of interest in the comparison was $p_{\text{out}}$.

Most of these schemes were designed assuming that each CR user is equipped with multiple transceivers, which may not be true for low-cost CR systems. In addition, these schemes are often based on a purely greedy strategy of using the best idle channel for a given CR transmission\textsuperscript{1}. As reported in [5], employing a greedy strategy in a CRN may result in a significant reduction in the achievable network throughput. Therefore, new distributed channel assignment algorithms and MAC protocols are needed for single-transceiver CRNs. These solutions should provide a better spectrum utilization.

### 1.2 Contribution

In this paper, we first investigate the spectrum access problem under hardware, interference, maximum transmission power, and received SINR constraints. Our design goal is to maximize the sum-rate achieved by all CR users, with respect to both channel assignment and transmission rate. The transmission rate in our setup depends on the PHY-layer implementation. Hence, unlike the case in previous works, our treatment is not restricted to the information-theoretic capacity\textsuperscript{2}, and can be applied to any arbitrarily given rate-SINR function. Specifically, the contribution of this paper is as follows. We first show that the joint rate/power control and channel assignment problem can be formulated as a mixed integer nonlinear programming (MINLP). To make this formulation more amenable for further processing, we note that actual communication systems only have a finite number of available channels, each associated with a maximum transmission power. Based on this fact, we transfer our MINLP to an integer linear programming problem (ILP) that only contains binary variables (i.e., we transfer our problem to rate/channel selection problem). This transformation applies to any arbitrarily given rate-SINR relationship. Then, we show that this ILP has a unimodular constraint matrix, and hence it can be optimally solved in polynomial time. Finally, we present a decentralized channel access mechanism that realizes the optimal channel assignment in a distributed manner.

To evaluate the performance of our proposed spectrum access mechanism, we conduct simulations over a CRN assuming a Rayleigh fading channel model. Our simulation results show that compared with the typical greedy mechanism, our mechanism significantly improves network throughput by up to 35% and preserves fairness.

\textsuperscript{1} The best channel is often defined as the one that supports the highest rate. We refer to this approach as the greedy approach.

\textsuperscript{2} The information-theoretic capacity is a logarithmic function of the received signal-to-interference-plus-noise ratio (SINR).
1.3 Organization

The rest of the paper is organized as follows. The system model is presented in Section 2. In Section 3, we formulate and solve the optimal channel assignment problem. The proposed spectrum access mechanism that employs the optimal assignment is described in Section 4. Section 5 presents the simulation results comparing our proposed solution with a baseline mechanism. Section 6 concludes the paper.

2 Network Model

We consider a distributed CRN that coexists with \( M \) licensed PRNs over a finite area. The PRNs are licensed on different, non-overlapping frequency bands. We assume that all the PRN bands have the same Fourier bandwidth \( BW \). In reality, a PRN may occupy multiple, non-contiguous, unequal frequency bands. Such a PRN can be easily represented in our setup by using multiple equal-bandwidth virtual PRNs, each operating over its own carrier frequency. For the \( i \)th PRN, we denote its carrier frequency by \( f_i \).

Each CR user is equipped with one half-duplex transceiver, and can only transmit (or receive) on one channel only. Without loss of generality, we assume that the bandwidth \( BW \) of a channel is sufficient to support at least one CR transmission. This is an acceptable assumption in many wireless systems that are built to operate in the unlicensed bands, including IEEE 802.11/a/b/g-compliant devices. To avoid corrupting the transmissions of licensed PR users, CR users continuously identify potential spectrum holes (idle PR channels) and opportunistically exploit them for their transmissions\(^3\).

Within a given neighborhood, multiple CR transmissions may contend for access to one of the available channels. Let \( \mathcal{C} \) and \( \mathcal{N} \) respectively denote the set of all \( C \) idle channels and the set of all CR transmission requests in a given neighborhood at a given time. We say the \( j \)th CR transmission \((j \in \mathcal{N})\) is successful if the following condition is met: “It is possible to find an idle channel \( i \) from the set \( \mathcal{C} \) such that the received SINR of the selected channel \( i \) \((\text{SINR}_{j}^{(i)})\) is greater than the SINR threshold \((\mu^*_j)\) that is required at the CR receiver of the \( j \)th transmission to achieve a target bit error rate over channel \( i \).” Formally, for transmission \( j \) and channel \( i \), the transmission rate, denoted by \( R_{j}^{(i)} \), \( \forall j \in \mathcal{N} \)

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\[^3\] The FCC recently adopted three principal methods that can be used to determine the list of idle channels that is potentially available for CR transmissions at a given time and in a given geographical location [18].
and \( i \in \mathcal{C} \), is obtained as follows:

\[
R_j^{(i)} = \begin{cases} 
  f(\text{SINR}_j^{(i)}) \text{ Mbps}, & \text{if } \text{SINR}_j^{(i)} \geq \mu_i^* \\
  0, & \text{otherwise.}
\end{cases}
\]  

(1)

where \( f(.) \) is any arbitrary rate-SINR function (e.g., Shannon’s equation, staircase function, etc.) that is decided by the PHY-layer implementation. It is worth mentioning that, in practice, this rate-SINR function takes the shape of a staircase, as shown in Figure 1.

Finally, to resolve collision between CR transmissions, we enforce an exclusive channel occupancy policy on CR transmissions, whereby a channel \( i \) assigned to a CR transmission \( j \) cannot be simultaneously assigned to another CR transmission in the same vicinity (inline with the CSMA/CA mechanism).

3 Optimal Channel Assignment

3.1 Maximum Sum-rate Single-transceiver Problem

Given the set of CR transmission requests \( \mathcal{N} \), the set of idle channels \( \mathcal{C} \), the maximum transmission powers over idle channels, our objective is to maximize the sum of rate of all CR transmissions over all channels in the current snapshot, subject to the following constraints:

a. A CR transmitter (receiver) can transmit (receive) over only one channel at a given time.

b. An idle channel cannot be assigned to more than one CR transmission in the same locality.
c. A CR user can potentially access channel $i$ if-and-only-if the received SINR over channel $i$ exceeds the prespecified threshold $\mu^*_i$. 

d. For a CR transmission $j$ and idle channel $i$, the transmission power $P_j^{(i)}$ is limited to $P_{\text{max}}^{(i)}$. If channel $i$ is occupied by a PR user, $P_j^{(i)} = 0$, where $P_{\text{max}}^{(i)}$ is the smaller of the FCC regulatory maximum transmission power over channel $i$ and the maximum power supported by the CR’s battery. 

**Observation 1:** In general, the transmission rate is an increasing function of the transmission power. Thus, for a CR transmission $j$ and a channel $i$, the maximum achievable transmission rate is achieved when transmission $j$ uses the maximum possible transmit power ($P_{\text{max}}^{(i)}$) over that channel.

### 3.2 Problem Formulation

Our objective is to maximize the sum of rate of all CR transmissions over all channels in current snapshot, by the means of optimal channel/rate assignment. Before formulating the problem, we define a binary variable $x_j^{(i)}$ as follows:

$$
x_j^{(i)} = \begin{cases} 
1, & \text{if channel } i \text{ is assigned to transmission } j \\
0, & \text{otherwise.} 
\end{cases} 
$$

Hence, the maximum sum-rate single-transceiver problem is stated as follows:

$$
\text{Maximize } \sum_{j \in N} \sum_{i \in C} R_j^{(i)} x_j^{(i)} \\
\text{Subject to } \\
\sum_{j \in N} x_j^{(i)} \leq 1, \quad \forall i \in C \\
\sum_{i \in C} x_j^{(i)} \leq 1, \quad \forall j \in N \\
\text{SINR}_j^{(i)} - \mu^*_i \geq (x_j^{(i)} - 1) \Gamma \\
0 \leq P_j^{(i)} \leq P_{\text{max}}^{(i)}, \quad \forall i \in C \text{ and } \forall j \in N
$$

where $\Gamma$ is a relatively large constant $\gg 1$ and $P_j^{(i)}$ is the transmit power of the $j$th transmission over channel $i$. Recall that SINR$_j^{(i)}$ is a function of $P_j^{(i)}$. 


### 3.3 Solution

An observation of the objective function of (3) and the constraints shows that this formulation constitutes a mixed integer nonlinear programming (MINLP) problem. The solution to such a problem is NP-hard, in general. However, for a finite number of idle channels and prespecified maximum transmission powers over various channels, a CR transmission can compute the maximum achievable rates \( r_j^{(i)} \) over each idle channel (see Observation 1). For all \( \text{SINR}_j^{(i)} < \mu_i^* \), where \( j \in \mathcal{N} \) and \( i \in \mathcal{C} \), we set \( r_j^{(i)} = 0 \). Given \( r_j^{(i)}, \forall j \in \mathcal{N} \) and \( i \in \mathcal{C} \), the MINLP problem can be transformed into the following binary linear programming (BLP) problem:

\[
\begin{align*}
\text{Maximize} & \quad \sum_{j \in \mathcal{N}} \sum_{i \in \mathcal{C}} r_j^{(i)} x_j^{(i)} \\
\text{Subject to} & \quad \sum_{j \in \mathcal{N}} x_j^{(i)} \leq 1, \quad \forall i \in \mathcal{C} \\
& \quad \sum_{i \in \mathcal{C}} x_j^{(i)} \leq 1, \quad \forall j \in \mathcal{N}.
\end{align*}
\]

(4)

Let \( \mathbf{A} \) and \( \mathbf{b} \) denote the linear constraint matrix and the right hand side of (4), respectively.

An examination of (4) reveals that its objective function is a linear function, the constraints are linear, \( \mathbf{A} \) is unimodular, and \( \mathbf{b} \) is integer (refer to the appendix for the concepts of unimodularity and the proof of \( \mathbf{A} \)'s unimodularity). Because of the unimodularity property, the optimal solution of the linear program (LP) relaxation is the optimal solution of (4) [19]. Therefore, the optimal solution of the BIP in (4) can be exactly found in polynomial-time using LP.

**Remark 1:** According to our treatment, CR users adopt a binary-level transmission power strategy, in which the transmission power over channel \( i, \forall i \in \mathcal{M} \) is given by:

\[
P_t^{(i)} = \begin{cases} 
P_{\text{max}}^{(i)}, & \text{if channel } i \text{ is idle;} \\ 
0, & \text{if channel } i \text{ is occupied by a PR user.}
\end{cases}
\]

(5)

It is worth mentioning that most of previously proposed spectrum access/sharing protocols and algorithms for CRNs were designed assuming the binary-level transmission power strategy given in (5) (e.g., [12, 14, 20–23]). This strategy ensures a non-overlapping (collision-free) channel occupancy between CR and PR users. ■
4 Proposed Channel Access Mechanism

4.1 overview

Based on the optimal channel assignment algorithm presented in Section 3, we now describe the proposed channel access protocol for CRNs. This protocol uses contention-based handshaking for exchanging control information. Our protocol is designed to (1) ensure exclusive channel occupancy policy (i.e., non-overlapping local channel occupancy between CR users) and (2) realize the optimal channel assignment in a distributed manner. To facilitate control packet exchange, we assume the availability of a prespecified control channel. Such a channel is not necessarily dedicated to the CRN. It may, for example, be one of the unlicensed ISM bands. Note that the existence of a common control channel is a characteristic of many MAC protocols proposed for CRNs (e.g., [5, 6, 12, 24, 3]).

4.2 Channel Contention

Admission Control Phase According to the optimal channel assignment, the set of idle channels $C$ and the instantaneous SINR information of all contending CR users in a given neighborhood should be known to all CR users in that neighborhood before assigning channels and transmission rates. Given the set $C$, the issue of announcing the SINR information of all contending users can be handled during the “admission phase” by introducing a contention period known as the *access window* (AW). The AW consists of $C$ fixed-duration access slots (AS), where the size of the AW is dynamically adjusted based on the number of available channels $C$ (Figure 2 illustrates the concept of AW admission control). A series of control packet exchanges take place during these slots, in which communicating CR users announce their instantaneous SINR information. It is worth mentioning that the use of an AW for contention was previously proposed in MACA-P and POWMAC protocols [25, 26]. However, in both protocols, the objective was not to facilitate the channel assignment mechanism, but to resolve collisions between control and data packets and address single-channel transmission power control.

Operation Details A CR user that has packets to transmit and that is not aware of any already established AW in its neighborhood can asynchronously initiate an AW. Each AS consists of the sum of an RTS duration, a CTS duration, and a maximum backoff interval. Control packets are sent at the maximum (known) transmission power imposed on the control channel. Upon receiving an RTS packet from a CR user, say $A$, that is initiating an AW, other CR users in the network synchronize their time reference with $A$’s AW. Suppose that a CR user $D$ is aware of $A$’s AW, and has a data packet to transmit. $D$ contends for the control channel in the next AS of $D$’s AW as follows. To prevent synchronized
Fig. 2. Example that illustrates the concept of dynamic AW in CRNs with $C = C_1$.

RTS attempts, $D$ first backs off for a random duration of time ($T$) that is uniformly distributed in the interval $[0, T_{\text{max}}]$; $T_{\text{max}}$ is a system-wide backoff counter. After this backoff time and if no carrier is sensed over the control channel, user $C$ sends its control packet in the current AS. After all the control packets have been exchanged, the optimal channel assignment/rate allocation of Section 3 is executed at every communicating CR user. In the rest of this paper, we refer to the channel access mechanism that uses our optimal assignment as OPT-MAC.

5 Performance Evaluation

We compare the performance of our proposed protocol with the CRN MAC (COMAC) protocol presented in [3]. COMAC is a CSMA-based MAC protocol for CRNs with a common control channel that employs a greedy channel assignment strategy of using the best idle channel for a given CR transmission. In our evaluation, we study the network performance under two different rate-SINR relationships: the information-theoretic capacity (i.e., Shannon’s equation) and a staircase-like function\(^4\) similar to the one shown in Figure 1.

Our results are based on simulation experiments conducted using CSIM programs (CSIM is a C-based process-oriented discrete-event simulation package [27]). Our performance metrics are the CRN sum-rate, and the fairness. We use the fairness index in [28] to quantify the fairness of a scheme according to the sum-rate of all CR users in the network. A Rayleigh fading signal propagation model to describe the channel gain between any two CR users is considered. Specifically, for a transmitter-receiver separation $d$, the received power over the $i$th channel is given by:

$$P_r(i) = P_o(i) \left(\frac{d}{d_o(i)}\right)^{-\xi(i)}, \quad d \geq d_o(i)$$

\[^4\] A staircase-like rate-SINR function typically characterizes practical multi-rate systems.
where $P_o^{(i)} = \frac{P_t^{(i)} G_t^{(i)} G_r^{(i)}}{(4\pi d_o^{(i)})^2}$ is the path loss of the close-in distance $d_o^{(i)} = \max\{\frac{2D^2}{l_i}, D, l_i\}$, $D$ is the antenna length, $P_t^{(i)}$ is the transmission power, $G_t^{(i)}$ is the antenna gain at the transmitter, $G_r^{(i)}$ is the antenna gain at the receiver, $l_i$ is the wavelength of $f_i$, $n$ is the path loss exponent, and $\xi^{(i)}$ is a normalized random variable that represents the power gain of the fading process. For Rayleigh fading, $\xi^{(i)}$ is exponentially distributed; $\Pr(\xi^{(i)} \leq y) = 1 - e^{-y}$ [20].

<table>
<thead>
<tr>
<th>Rate Index</th>
<th>Rate</th>
<th>SINR_{db}</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1 Mbps</td>
<td>1</td>
</tr>
<tr>
<td>R2</td>
<td>4.5 Mbps</td>
<td>5</td>
</tr>
<tr>
<td>R3</td>
<td>6 Mbps</td>
<td>7</td>
</tr>
<tr>
<td>R4</td>
<td>7 Mbps</td>
<td>10</td>
</tr>
<tr>
<td>R5</td>
<td>8.5 Mbps</td>
<td>15</td>
</tr>
<tr>
<td>R6</td>
<td>9.5 Mbps</td>
<td>20</td>
</tr>
<tr>
<td>R7</td>
<td>10.5 Mbps</td>
<td>25</td>
</tr>
<tr>
<td>R8</td>
<td>11 Mbps</td>
<td>31</td>
</tr>
</tbody>
</table>

Table 1. Rate-SINR relationship

5.1 Simulation Setup

We consider $N$ pairs of one-hop transmitter-receiver CR users in a 100 meter × 100 meter area. To simplify our simulation, we assume that all CR users are within the transmission range of each other, so that any control packet sent from a CR can be heard by all other CR users. The locations of the CR transmitters and receivers are randomly assigned within the simulation region. We assume that there are $M = 11$ channels, each has 2.5 MHz of bandwidth and licensed to one PRN. The carrier frequency of the $i$th PRN is $f_i = 900 + i$ MHz, for $i = 1, \ldots, M$. CR users can opportunistically access the 11 channels. We set $\mu^*_i = \mu^*$, $\forall i \in M$. For the simulated staircase rate-SINR function, the values of the different supported rates and their corresponding SINR thresholds are summarized in Table 1. Note that for both the information-theoretic and the staircase rate-SINR functions, the achieved rate is 0 if the received SINR < $\mu^*$. We divide the time into slots, each of length 100 ms. A 2-state Markov model that alternates between two states: IDLE and BUSY is considered to determine the status of a PR channel at any given time. A BUSY (IDLE) state indicates that some (no) PR user is transmitting over the given channel. For channel $i$, denote the average IDLE and BUSY durations of the PR activity by $\lambda_i$ and $\mu_i$, respectively. At any given time slot, the $i$th PRN is active with probability $P_B^{(i)} = \frac{\lambda_i}{\lambda_i + \mu_i}$. We set $\mu_i = 100$ ms and $\lambda_i = \lambda, \forall i \in \{1, \ldots, M\}$. Accordingly, $P_B^{(i)} = P_B, \forall i \in M$. The locations of the CR transmitters and receivers are randomly assigned within the simulation region. We set the maximum transmission power of a CR user to $P_{\text{max}} = 1$ W, the thermal noise power density to $10^{-21}$ W/Hz for all channels, the path loss exponent to $n = 4$, and the antenna
length to $D = 5$ cm. The results presented below are based on the average of 30 randomly generated topologies, with a simulation time of 5000 time slots for each topology.

<table>
<thead>
<tr>
<th>Number of CR Links (N)</th>
<th>CRN Sum-rate (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>OPT-MAC, $\mu^* = 3$</td>
</tr>
<tr>
<td>4</td>
<td>OPT-MAC, $\mu^* = 9$</td>
</tr>
<tr>
<td>6</td>
<td>COMAC, $\mu^* = 3$</td>
</tr>
<tr>
<td>8</td>
<td>COMAC, $\mu^* = 9$</td>
</tr>
</tbody>
</table>

(a) Assuming Shannon Capacity formula

(b) Assuming staircase formula

**Fig. 3.** CRN sum-rate vs. number of CR links $N$.

### 5.2 Simulation Results

We first compare the achieved sum-rate of all CR links for both OPT-MAC and COMAC protocols. The CRN sum-rate is plotted as a function of the number of active CR links in Figure 3. In Figure 3(a) (Figure 3(b)), we consider Shannon’s equation (the staircase function) to compute the achieved rates over idle channels for each active CR link.
Figure 3 shows that as the number of CR users increases, OPT-MAC significantly outperforms COMAC, irrespective
of the employed rate-SINR relationship. Specifically, OPT-MAC improves the overall achieved sum-rate by up to 35%
compared to COMAC. This improvement is mostly attributed to the employed proper channel assignment. Figure 5 shows
the achievable sum-rate as a function of PR activities ($P_B$). Figure 5 reveals that the throughput gain of OPT-MAC over
COMAC is smaller at larger $P_B$. This is expected since the larger the value of $P_B$, the lower the chances of finding idle
channels, which consequently reduces the performance gain (similar behavior was observed when stair-case rate-SINR
function is used).

In Figure 6, we study the impact of the channel assignment strategy on fairness. It is clear that both protocols achieve
comparable fairness. This can be attributed to the fact that, in both protocols, CR users contend over the control channel
using a variant of the CSMA/CA mechanism. Finally, Figure 7 depicts the channel usage, defined as the fraction of time
in which a specific channel is used for CR transmissions. For both protocols, channel usage is roughly evenly distributed
among all channels, irrespective of the number of CR links.

6 Conclusions

In this paper, we proposed a solution to solve the joint rate control and channel assignment problem for the coordinated
channel access in opportunistic CRNs. Our solution improves the CRN throughput through an optimal channel assignment
process. The assignment process dynamically assigns channels to CR transmissions with the objective of maximizing the
achieved CRN sum-rate and subject to interference and SINR constraints. In order to realize our solution in a distributed
manner, we developed a novel CSMA/CA-based MAC protocol for CRNs with access window (AW). The purpose of the
AW is to announce CR transmission requests and SINR information, and resolve contention between CR transmissions.
We compared the performance of our MAC protocol with that of a reference multi-channel CSMA/CA CRN MAC
(COMAC) protocol that is based on a greedy channel assignment. We showed that our protocol achieves about 35%
increase in throughput over the COMAC protocol, while preserves fairness. In summary, our protocol provides better
spectrum utilization by achieving a larger network throughput.
Appendix

Total Unimodularity of the Constraint Matrix

Definition 1 A matrix $B$ is said to be totally unimodular if the determinant of every square submatrix (minor) of $B$ is in the set $\{-1, 0, 1\}$ [30]. The following theorem provides sufficient but not necessary conditions for a matrix of 0’s and 1’s to be totally unimodular.

Theorem 1 An $m \times n$ matrix $B$ is totally unimodular if its rows can be partitioned into two disjoint matrices $B_1$ and $B_2$, with the following properties [30]:

- Every entry in $B$ is 0 or 1.
- Every column of $B$ contains at most two 1’s.
- Every column of $B_1$ and $B_2$ contains at most one non-zero element.

Theorem 2 The constraint matrix $A$ is totally unimodular.

Proof 1 To prove this, we partition the matrix $A$ into two disjoint matrices $A_1$ ($N \times CN$) and $A_2$ ($C \times CN$), (i.e.,

$$A = \begin{bmatrix} A_1 & A_2 \end{bmatrix}^T,$$

where $A_1$ and $A_2$ correspond to the first and second constraint in (4), respectively. The matrices $A_1$ and $A_2$ can be written as:

$$A_1 = \begin{bmatrix} C \\ 1 \ldots 1 \ldots 0 \ldots 0 \ldots 0 \ldots 0 \\ \vdots \\ 0 \ldots 0 \ldots 1 \ldots 1 \ldots 0 \ldots 0 \ldots 0 \\ \vdots \\ 0 \ldots 0 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \\ C \end{bmatrix}_{N \times CN}$$ (7)

and

$$A_2 = \begin{bmatrix} C \\ 10 \ldots 0 \ldots 0 \ldots 0 \ldots 0 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \\ \vdots \\ 010 \ldots 0 \ldots 010 \ldots 0 \ldots 010 \ldots 0 \\ \vdots \\ 0 \ldots 1 \ldots 0 \ldots 1 \ldots 0 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \ldots 1 \\ C \end{bmatrix}_{C \times CN}$$ (8)
Note that $A, A_1, \text{ and } A_2$ satisfy all the sufficient conditions of total unimodularity given in Theorem 1. Therefore $A$ is totally unimodular.

References

(a) Assuming Shannon Capacity formula

(b) Assuming stair-case formula

Fig. 4. CRN sum-rate vs. SINR threshold, assuming Shannon formula.
Fig. 5. CRN sum-rate vs. $P_B$, assuming Shannon capacity formula.

Fig. 6. Fairness index vs. number of CR links ($N$) for $\mu^* = 3$ (other values of $\mu^*$ depicted similar behavior).

Fig. 7. Channel usage (%) for $\mu^* = 3$ and $N = 18$ (other values of $\mu^*$ depicted similar behavior).