Experimental and Theoretical Evidence for the Existence of Absolute Acoustic Band Gaps in Two-Dimensional Solid Phononic Crystals

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Experimental measurements of acoustic transmission through a solid-solid two-dimensional binary-composite medium constituted of a triangular array of parallel circular steel cylinders in an epoxy matrix are reported. Attention is restricted to propagation of elastic waves perpendicular to the cylinders. Measured transmitted spectra demonstrate the existence of absolute stop bands, i.e., band gaps independent of the direction of propagation in the plane perpendicular to the cylinders. Theoretical calculations of the band structure and transmission spectra using the plane wave expansion and the finite difference time domain methods support unambiguously the absolute nature of the observed band gaps.

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Optical properties of heterogeneous materials with a periodic structure have received a great deal of attention during the past two decades. Much effort has focused on the search of large band gaps in the optical band structure of periodic inhomogeneous dielectric materials. Several geometries of these so-called "photonic" crystals have been proposed: one-dimensional (1D) systems in the form of Bragg lattices [1] or comblike structures [2], 2D systems such as arrays of parallel cylinders embedded in a matrix, and 3D crystals with various distributions of spherical inclusions suspended in a host matrix [3]. Recently, the mathematical analogy between Maxwell's equations and the equations of linear elasticity has spurred a "renewed interest" in "phononic" crystals, that is periodic inhomogeneous elastic media exhibiting forbidden bands in their acoustic transmission spectrum. In these materials, the density and the elastic constants are periodic functions of the position. Although the propagation of elastic waves in periodic composite materials is an old topic in condensed matter physics and/or acoustics [4], the search of acoustic band gaps in heterogeneous materials gave rise, in recent years, to numerous theoretical and experimental investigations. Theoretical models of 2D [5,6] and 3D [7,8] phononic crystals based on the plane wave expansion (PWE) method have shown that the width of the acoustic band gaps strongly depends on the composition and the geometry as well as on the nature of the constitutive materials. A large contrast in physical properties between the inclusions and the host material is required to obtain large acoustic band gaps [9]. More recently, Sigalas *et al.* [10,11] applied the finite difference time domain (FDTD) method [12], well known in the field of photonic crystals [13,14], to the study of two- and three-dimensional elastic band gap materials. In contrast to the PWE method, the FDTD method enables the calculation of the acoustic transmission coefficient of a finite composite sample that can be measured routinely in experiments. Moreover, the FDTD method can be applied to mixed (solid-fluid) composites where the PWE fails to work [15] due to the vanishing of the shear modulus in the fluid component. In the FDTD method, the elastic wave equations are discretized in both the spatial and the time domains with appropriate boundary conditions. The evolution of the elastic displacement field is calculated in the time domain and when Fourier transformed results in the acoustic transmission spectrum. The agreement between the FDTD and the PWE methods seems to be excellent in locating the forbidden bands [10].

Recently, we investigated experimentally the transmission of elastic waves in 2D solid composite media composed of square and centered rectangular arrays of Duralumin cylindrical inclusions in an epoxy resin matrix [16]. In these cases, the measured transmission drops to noise level throughout frequency intervals in reasonable agreement with the forbidden frequency bands calculated with the PWE method. In parallel, other groups have studied acoustic band gaps in mixed binary 2D composite materials such as Hg, oil, or air cylinders in an Al matrix [11,17] or metallic rods in air [18–20].

In this Letter, we present a combined theoretical and experimental study of a triangular 2D solid-solid phononic crystal that demonstrates unambiguously the existence of absolute acoustic band gaps. Let us recall that of all phononic crystals studied to date theoretically, 2D triangular lattices of parallel cylinders in a matrix are expected to exhibit the largest gaps [9]. We compute the band structure for elastic waves propagating into the plane perpendicular to the cylinders and the acoustic transmission spectrum for longitudinal waves with the PWE method and the FDTD scheme, respectively. The numerical results are compared with transmissions measured experimentally with two

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finite size composite samples oriented along two different directions of propagation.

We are dealing with a triangular array of steel cylinders embedded in an epoxy resin matrix. The choice of these materials is based on the strong contrast in their densities and elastic constants [21]. We have manufactured two samples of the same physical dimensions, 85 mm × 85 mm × 29 mm. The steel cylinders have a diameter $d = 4$ mm. The periodicity of the triangular lattice is $a = 6.023$ mm. This results in a filling fraction $f = \pi d^2/2a^2\sqrt{3} = 0.4$. In order to analyze the effect of the direction of propagation on the band gaps, the first sample contains 60 scatterers arranged in such a manner that its thickness is parallel to the ΓJ direction of the triangular Brillouin zone. A second sample consists of 67 cylinders with the sample thickness along the ΓX direction. We have illustrated in Figs. 1(a) and 1(b) the two-dimensional cross sections of these samples. Our experimental setup is based on the well-known ultrasonic transmission technique which uses a couple of ultrasonic broadband transmitter/receiver transducers with a central frequency on the order of 1 MHz and a diameter of 25 mm (Panametrics contact transducers type Videoscan No. V102). A pulser/receiver generator (Panametrics model 5052 PR) produces a short duration (about 100 ns) large amplitude (200 to 380 V) pulse which is applied to the transmitting transducer launching the probing longitudinal waves. The signal acquired by the receiver is postamplified and then digitized with a maximum sampling rate of 100 MHz (or 10 ns/point) by a Lecroy digital oscilloscope. To reduce the random errors, each measurement is averaged over a sample size of 200 with the oscilloscope which performs fast Fourier transform on the acquired signals. Both emitter and receiver are coupled to the transversal walls of the specimen using a coupling gel. The acoustic transmission spectra of Figs. 2(a) and 2(b) clearly show two forbidden bands. The first band gap appears between 120 and 270 kHz for the two directions considered. At higher frequencies, the transmission drops to noise level between 350 and 510 kHz in the direction ΓJ and 430 to 560 kHz in the direction ΓX. The intersection frequency domain 430–510 kHz is therefore independent of the direction of propagation. One should stress that because of the strong attenuation of the transmitted power at these high frequencies, it is quite difficult to define precisely the edges of the region with noise level transmission.

We have calculated with the PWE method, the band structure of an infinite periodic triangular lattice with the same lattice parameter and physical characteristics as the experimental samples. We limit the wave propagation to the plane XY perpendicular to the cylinders. This approximation has for effect to decouple the elastic displacements in the XY plane (called XY modes) and those parallel to the Z direction denoted Z modes [see Fig. 1 for an illustration of the (O, X, Y, Z) reference frame]. In the PWE method, the 2D periodicity in the XY plane allows one to develop the density and the elastic constants in the Fourier series.

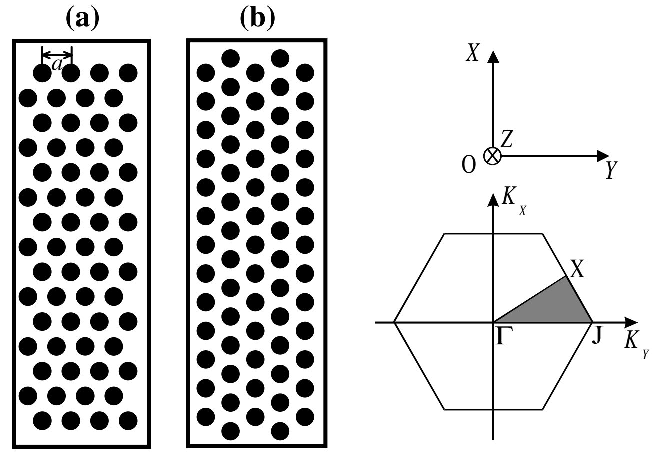

FIG. 1. Two-dimensional cross sections of the triangular array of steel cylinders embedded in an epoxy matrix: (a) the "ΓJ" sample and (b) the "ΓX" sample. The steel cylinders, of circular cross section, are parallel to the Z axis of the Cartesian coordinate system $(0, X, Y, Z)$. The lattice parameter a is defined as the distance between two nearest neighboring cylinders. The inset shows the irreducible Brillouin zone of the triangular array.

Then, the equations of linear elasticity become standard eigenvalue equations for which the size of the matrices involved depends on the number of $\vec{G}$ vectors of the reciprocal lattice taken into account in the Fourier series [6]. The band structure for the XY modes (Fig. 3) was calculated here with 1381 $\vec{G}$ vectors. This number of $\vec{G}$ vectors insures sufficient convergence of the Fourier series. Figure 3 indicates the existence of two absolute band gaps extending throughout the 2D triangular Brillouin zone. The first gap extends from the frequency of 124 kHz up to 276 kHz and is in very good agreement with the experiment. The second absolute gap occurs between 441 and 483 kHz and

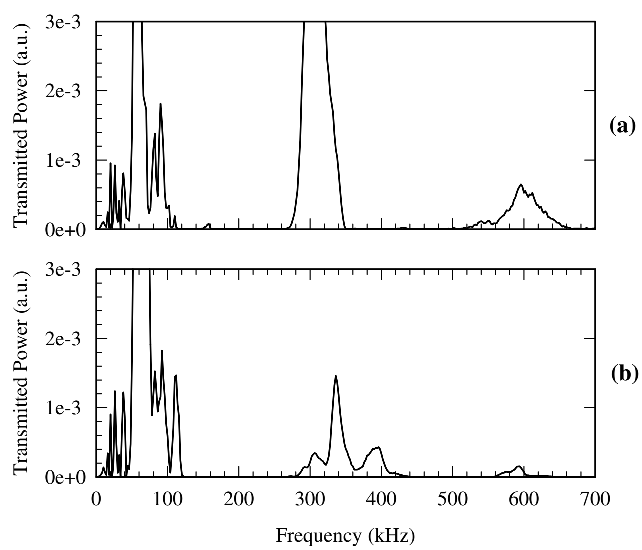

FIG. 2. Transmission power spectrum measured perpendicular to the vertical faces of the (a) "ΓJ" sample and (b) "ΓX" sample. The transmitted power is given in arbitrary units. The probing signal is a longitudinal wave.

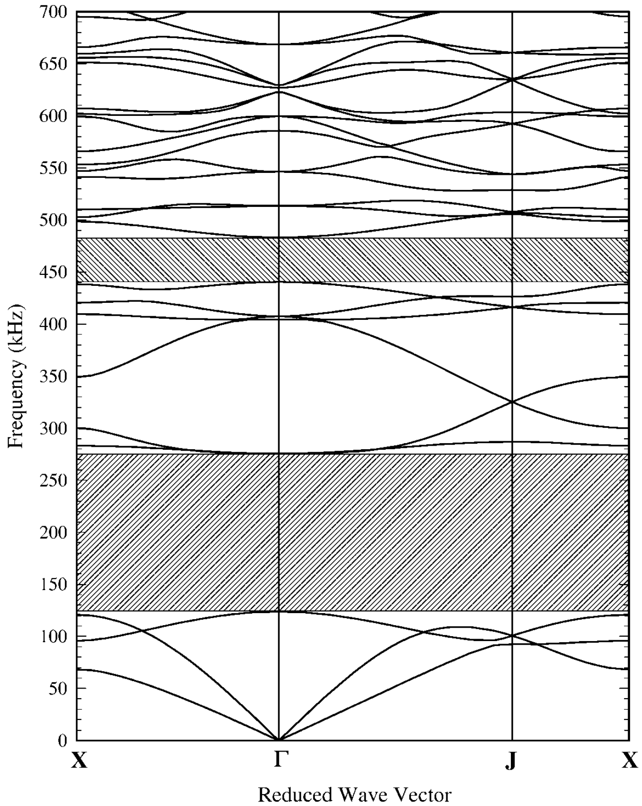

FIG. 3. PWE results for the band structure of the two-dimensional XY modes of vibration in the periodic triangular array of steel cylinders in an epoxy resin matrix for a filling fraction $f = 0.4$. The reduced wave vector $\vec{k}(k_X, k_Y)$ is defined as $\vec{K}a/2\pi$ where $\vec{K}$ is a two-dimensional wave vector. The points Γ, J, and X are defined in the inset of Fig. 1. Absolute band gaps are represented as hatched areas.

falls also in the experimental range although, for both ΓJ and ΓX directions, the transmission drops to the noise level throughout a wider range than the one predicted by the band structure. This difference can be attributed to the low level of transmission in this range of frequency, but more especially to the fact that some eigenfrequencies of the structure may not contribute very significantly to the transmission.

To gain a better insight into these effects, and also to investigate the qualitative behavior of the transmission inside the passbands, we have applied the FDTD method to calculate the transmission through two finite size samples oriented, respectively, along the ΓJ and ΓX directions. We limit the calculations to a strictly 2D FDTD scheme. The FDTD samples are composed of three adjacent regions. The probing signal corresponding to a longitudinal wave that propagates along the Y direction (see Fig. 1) is launched from the first region and detected in the third one. This signal is the superposition of five sinusoidal waves of frequencies 100, 250, 400, 550, and 700 kHz weighed by a Gaussian profile of full width at half maximum of 13.5 mm. Transmission of this signal through a homogeneous epoxy medium produces a broadband spectrum whose envelope resembles the experimental one. The central region contains the phononic crystal. To probe the ΓX direction, the central region with a thickness (along the Y direction) of $3a\sqrt{3}$ and a width (along the X direction) of a contains nine cylinders. The ΓJ direction is modeled with a rectangular central region containing eight cylinders, of thickness, $4a$, and width, $a\sqrt{3}$. Periodic boundary conditions are applied in the X direction perpendicular to the direction of propagation. Absorbing boundary conditions are imposed on the external surfaces of the first and third regions. Space and time are discretized with fine enough intervals to achieve convergence of the algorithm. Three output signals (longitudinal vibrations) are detected at different locations. The transmission spectra are calculated as the averages of their Fourier transforms.

The FDTD computed spectrum in the ΓJ direction is shown in Fig. 4(a) where the longitudinal component, u_Y, of the displacement field is given in arbitrary units as a function of frequency. One observes an overall agreement with the experiment. The width of the first gap is well reproduced. The shape of the second passband as well as its width is in better accord compared to the band structure. Moreover one can notice a maximum of the transmission coefficient in the middle of the lowest passband as well as a qualitative similarity of the theoretical and experimental transmissions at higher frequencies.

Figure 4(b) contains the FDTD results in the ΓX direction. Again, the width of the first gap and the occurrence of a maximum of transmission in the middle of the lowest passband are in good agreement with the experiment. However some discrepancies remain when considering the location and shape of the second passband. The FDTD spectrum presents, between 260 and 400 kHz, two peaks separated by a narrow stop band centered around 320 kHz,

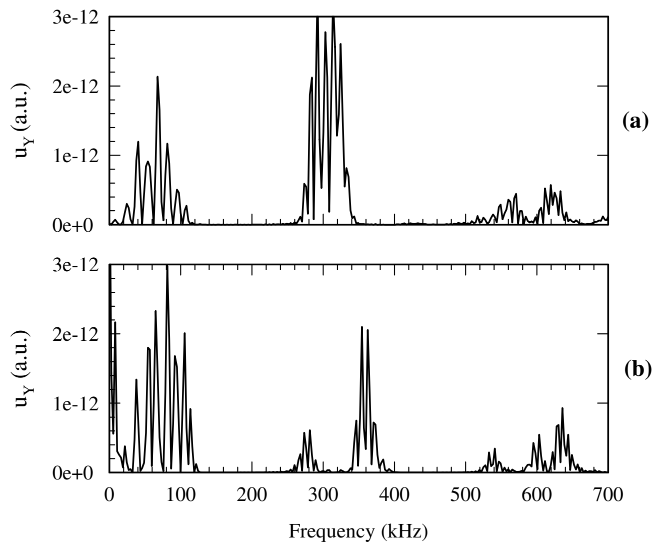

FIG. 4. Transmission spectrum computed with the FDTD method along the ΓJ direction (a) and along the ΓX direction (b) of propagation. The longitudinal component u_Y of the output displacement field is given in arbitrary units.

in accordance with the local gap in the band structure of Fig. 3. In contrast the experimental spectrum exhibits three distinct peaks, the second of maximal amplitude centered on 340 kHz. Also the second gap (430–560 kHz in Fig. 2) is located nearly 40 kHz higher than predicted by the FDTD method. At this point, it is worthwhile to notice that the measurement of sound velocities in epoxy is subject to important uncertainties (since these velocities are quite dependent on the conditions of sample preparation), and in turn the location of the higher bands in the theoretical calculations may be affected by a few tens of kHz. Therefore a detailed interpretation of the measured spectrum above 280 kHz, with a one to one correspondence between experiment and theory, does not seem possible although one may infer that there is a good correspondence between the major peak (around 340 kHz) in the measured and calculated spectra. Another point that deserves further attention concerns the divergence of the emitted acoustic signal, i.e., the fact that the input signal is not a plane wave but is composed of a set of wave vectors inside a cone around the incident direction. Therefore other modes than those considered till now in the PWE and FDTD calculations can be excited, that in turn may affect the final transmission.

In conclusion, we have investigated experimentally and theoretically the propagation of acoustic waves in a binary 2D phononic crystal constituted of a triangular array of parallel, circular, steel cylinders in an epoxy resin matrix. We have limited the wave propagation to the plane perpendicular to the cylinders. The measurements and the numerical calculations prove unambiguously the existence of two absolute stop bands independent of the direction of propagation of the acoustic waves. Besides the band gaps, one can establish some qualitative and even semiquantitative correspondences between the experimental and theoretical transmission spectra inside the pass bands. However, a more quantitative comparison would need to repeat such experiments with other samples (for instance to check the possibility of defects during the sample preparation, different thicknesses of the samples, etc.), and also to include in the FDTD calculation the possibility of three-dimensional propagation due to the divergence of the initial pulse. In this respect, an analysis of the eigenvectors associated with the different modes would be also helpful for an understanding of the details of the experimental transmission spectra.

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