Two-dimensional phononic crystal with tunable narrow pass band: Application to a waveguide with selective frequency

A. Khelif[a)]
Laboratoire de Physique et Métrologie des Oscillateurs, CNRS UPR 3203, 32 avenue de l'Observatoire, 25000 Besançon Cédex, France

P. A. Deymier
Department of Materials Science and Engineering, University of Arizona, Tucson, Arizona 85721

B. Djafari-Rouhani, J. O. Vasseur, and L. Dobrzynski
Laboratoire de Dynamique et Structures des Matériaux Moléculaires, UMR CNRS 8024, UFR de Physique, Université de Lille I, 59655 Villeneuve d'Ascq Cédex, France.

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We study theoretically the propagation of elastic waves in two-dimensional composite media composed of a square array of hollow steel cylinders embedded in water using the finite-difference time-domain method. These composite media constitute a class of acoustic band gap materials with narrow pass bands in their transmission stop bands. The frequency at which the pass band occurs is tunable by controlling the inner radius of the tubular steel inclusions. The effect of the tube inner radius on the transmission spectrum is semiquantitatively separable from the effect of the composite periodicity. A linear defect formed of a row of hollow cylinders in an array of filled cylinders produces an elastic waveguide that transmits at the narrow pass band frequency. We show that two of these tunable waveguides with different inner radii can be employed to filter and separate two specific frequencies from a broad band input signal.
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I. INTRODUCTION

Acoustic band gap materials (ABGs),[1–5] also called phononic crystals, are receiving increasing attention as potential materials for the design of elastic-acoustic wave guides[6,7] or filters. ABGs are composite elastic media, composed of two- or three-dimensional periodic repetitions of different solids or fluids, that exhibit stop bands in the spectrum of transmission of elastic waves. The existence, location, and width of acoustic band gaps in the transmission spectrum results from a large contrast in the value of the elastic constants and/or mass density of the constitutive materials. Guidance of the waves can be achieved by creating extended linear defects in an ABG, for instance, by removing a row of cylindrical inclusions in an originally periodic two-dimensional phononic crystal.[7] Grafting other defects (e.g., a side branch or stub) along an extended waveguide permits some frequency selectivity in the form of zero transmission in the primary transmission spectrum of the perfect guide.[6] However, for practical application, it is more desirable to achieve selectivity with narrow passing bands in guides that otherwise do not transmit elastic waves over some wide range of frequencies.

We report and describe a class of two-dimensional (2D) ABGs that incorporates tunable narrow passing bands in their band gaps. The frequency of the passing band is controlled by modifying the geometry of the cylindrical inclusions, with little change in the location and form of the transmission gap. This class of ABGs with tunable narrow pass bands (TNPBs) further provides a route means by which to design selective acoustic waveguides with potential filtering and demultiplexing capabilities. We study the properties of TNPB ABGs theoretically using the finite-difference time-domain (FDTD) method.[8] The FDTD method solves the elastic wave equation by discretizing time and space and replacing derivatives by finite differences. Properties of solid/solid and fluid/solid ABGs calculated with this method have been shown to compare very well with experimental measurements.[9,10]

II. TUNABLE NARROW PASS BANDS AND SELECTIVE FREQUENCY FILTER WAVEGUIDES

Our calculations of transmission coefficient and dispersion curves are performed for 2D phononic crystals of square geometry composed of hollow steel cylinders in a fluid (water) matrix [see Fig. 1(a)]. We limit the model to elastic displacements, velocities, and stress fields in the XY plane perpendicular to the cylindrical inclusions. To calculate the transmission coefficient, the model system is built of three parts. The finite phononic composite (composed of five periods in the direction of propagation, Y, and one period in direction X along which periodic boundary conditions are applied) occupies the center region. This geometry corresponds to the ΓX direction of the irreducible Brillouin zone of the square lattice. The composite region is sandwiched between two homogeneous parts composed of water. Absorbing Mur boundary conditions[11] are imposed at the free ends of the homogeneous regions. A broad band traveling wave packet is launched in the first homogeneous region.

[a)]Corresponding author; electronic mail: abdelkrim.khelif@lpmo.edu.fr

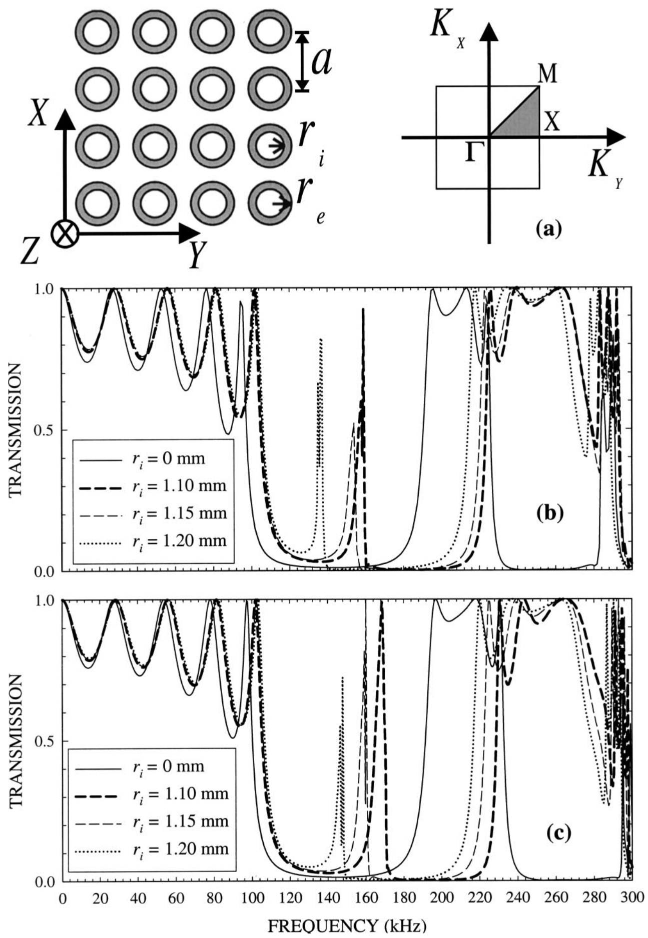

FIG. 1. (a) Two-dimensional cross section of a phononic crystal composed of a square array of hollow steel cylinders in water. The irreducible Brillouin zone of the square lattice is also presented. (b) Transmission spectra computed along the ΓX direction of the irreducible Brillouin zone of the square lattice of the filled cylinder ABG and three NPB ABGs with steel tube inner radii of $r_i = 1.10$, 1.15, and 1.20 mm. The lattice parameter and the steel tube outer radius are $a = 5$ mm and $r_e = 2$ mm, respectively. (c) Same as in (b) but the FDTD calculations were done by doubling the number of grid points and increasing the number of time steps.

The signal transmitted is recorded at the end of the second homogeneous region and integrated along its width. The Fourier transform of the transmitted signal normalized to the Fourier transform of a signal propagating through a homogeneous water system of the same physical dimensions as the model composite yields a transmission coefficient. The longitudinal and transverse speeds of sound in steel are taken as 5825 and 3226 m/s. The longitudinal speed of sound in water is 1490 m/s. The density of steel and water are 7.78 and 1 g/cm^3, respectively. The lattice parameter of the square lattice of cylinders is $a = 5$ mm. The inclusions are hollow cylinders with an outer radius of $r_e = 2$ mm and a variable inner radius, r_i. Figure 1(b) shows the transmission coefficient in the ΓX direction as a function of the frequency for several values of r_i. For $r_i = 0$, the inclusions are filled solid cylinders. In this case, the transmission spectrum exhibits a gap over the interval of frequency of $[100, 180]$ kHz. We have verified with a FDTD calculation of the transmission coefficient along the ΓM direction of the Brillouin zone that the stop band observed corresponds to an absolute gap, that is, its existence is independent of the direction of propagation. The models with $r_i > 0$ show transmission spectra that do not differ qualitatively from the filled cylinders at frequencies below the transmission gap. The lower limit of the stop band is not dependent on the inner radius and appears approximately 10 kHz above the filled cylinder case. The gap's upper frequency is more sensitive to the nature of the inclusions. Hollow tubes enlarge the stop band with upper frequencies ranging from 200 to 220 kHz for the inner radii studied. The upper edge of the band gap moves toward lower frequencies as the radius increases. The most remarkable feature, however, is the existence of a narrow pass band in the mostly unmodified band gap of the ABGs with hollow cylinders. As the inner radius increases, the frequency at which the narrow pass band occurs decreases. The occurrence of the narrow pass band in the stop band depends on the contrast in elastic constant and density of the inclusion and matrix constitutive materials. A previously studied high contrast ABG material, composed of hollow copper cylinders in air,[10] did not possess such a narrow pass band and its transmission spectrum was nearly identical to that of a system of filled cylinders. In the above calculations, the discretization mesh in real space is chosen to be $\Delta x = \Delta y = a/70$, whereas $N = 2^{19}$ steps of time were taken into account with $\Delta t = 0.25\Delta x/c\sqrt{2}$, where c is the longitudinal velocity of sound in steel. To check the accuracy of this calculation with regard to convergence of the results, we have repeated the same calculation by doubling the number of grid points in each direction and taking $N = 2^{21}$. The results presented in Fig. 1(c) show a slight increase in the frequencies of the narrow pass bands as well as in the frequencies of the upper edge of the gap. These results remain unchanged when the number of grid points is increased further to three times the initial value. Nevertheless for the sake of consistency and because of limited computational resources, all FDTD transmission spectra reported in the following were calculated with the same parameters as those in Fig. 1(b).

We have calculated the band structure of the NPB ABGs using the FDTD method.[12] The narrow pass band observed in the transmission spectrum results from the overlap of several branches in the dispersion curves (see Fig. 2). These nearly flat branches do not correspond to localized modes in the water filled cavity of the tubes. Replacing the water inside the tubes by several other fluids with longitudinal speed of sound and density that differ from that of water by nearly 20%–30% does not alter the NPB frequency.

The NPB ABGs qualitatively obey a principle of superposition and separability. That is, the transmission spectrum of a mixed ABG system composed of a square lattice of cylinders with different inner radii (same outer radii) in successive layers may be qualitatively constructed from the transmission spectrum of one of the ABGs with several NPBs inside the transmission gap. There is a NPB for each inner radius represented in the mixed system. The frequencies at which the NPBs appear are those of NPB ABGs with uniform arrangement of cylinders with the same inner radius. The arrangement of the different cylinders within the mixed ABG appears to impact the intensity of the NPB peaks relative to each other. The superposition and separability of the

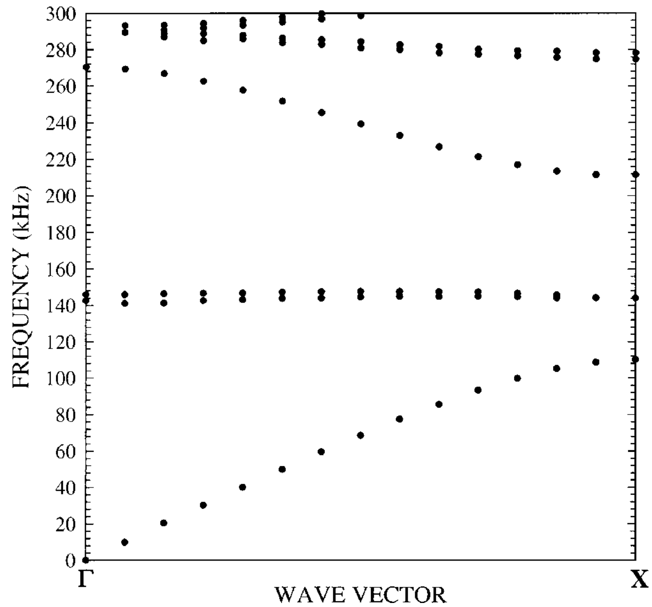

FIG. 2. Band structure of the NPB ABG with inner radius of 1.20 mm, computed with the FDTD method along the ΓX direction of the irreducible Brillouin zone of the square lattice.

NPBs inside the transmission gap in the NPB ABG studied here offer a unique opportunity for designing acoustic-elastic waveguides with well defined transmitting frequencies. To illustrate this concept we studied a linear waveguide in an ABG system composed of filled cylinders by substituting a row of filled cylinders with hollow cylinders. The center region of the FDTD model contains five rows of seven cylinders arranged on a square lattice ($a=5$ mm) in the Y direction parallel to wave propagation [Fig. 3(a)]. The third row is made up of hollow cylinders whereas all other cylinders are filled. The inner radius of the hollow cylinders is denoted as $r_i^{(1)}=1.2$ mm. Because of periodic boundary conditions in the X direction perpendicular to the guide, the system is effectively multilayered. However, the four rows of filled cylinders that separate each periodic waveguide are sufficient to prevent significant leakage between the guides. We probe the transmission of the waveguide by launching the same broad band signal as before on one of its sides and recording the displacement at the waveguide exit. The transmitted signals that exit the guide are Fourier transformed and normalized to the spectrum of homogeneous water. With this procedure the maximum value of transmission can exceed 1. In Fig. 3(b) we present the transmission spectrum of the hollow cylinder guide. This spectrum nearly reproduces that of the NPB ABG with cylinders having radius $r_i^{(1)}=1.2$ mm. This result shows that elastic waves with frequencies in the interval of 100–200 kHz cannot propagate into the waveguide, they are within the narrow frequency band centered on frequency of 148 kHz. We note that this frequency differs slightly from that of the NPB ABG due to the overlap of vibrational modes at the interface between the guide and the filled cylinders. We have also verified that transmission in nonrectilinear guides (e.g., a guide with a kink) also occurs readily. The selectivity of the NPB ABG can be used to separate specific frequencies from a broad band input signal and guide them to different locations. To that end we have investigated the behavior of two hollow cylinder waveguides in a filled cylinder ABG. The system is composed of 9 rows of 10 cylinders repeated periodically along the X direction. As seen in Fig. 4(a), filled cylinders in the third and seventh rows are replaced by hollow cylinders of inner radii $r_i^{(1)}=1.1$ mm and $r_i^{(2)}=1.2$ mm, respectively. A broad band signal is input on one side of the system. The two transmitted spectra are reported in Fig. 4(b). The two spectra match quite well over most frequencies but as expected the second guide transmits

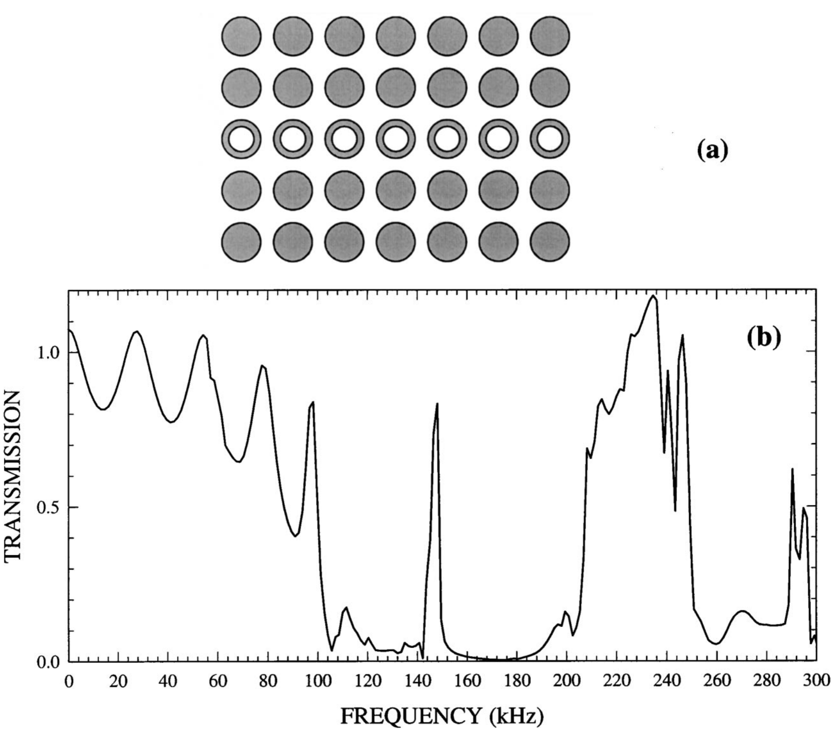

FIG. 3. (a) Two-dimensional cross section of a NPB waveguide with inner radius of 1.2 mm. (b) Transmission spectrum of the waveguide calculated at its exit for a broad band input signal.

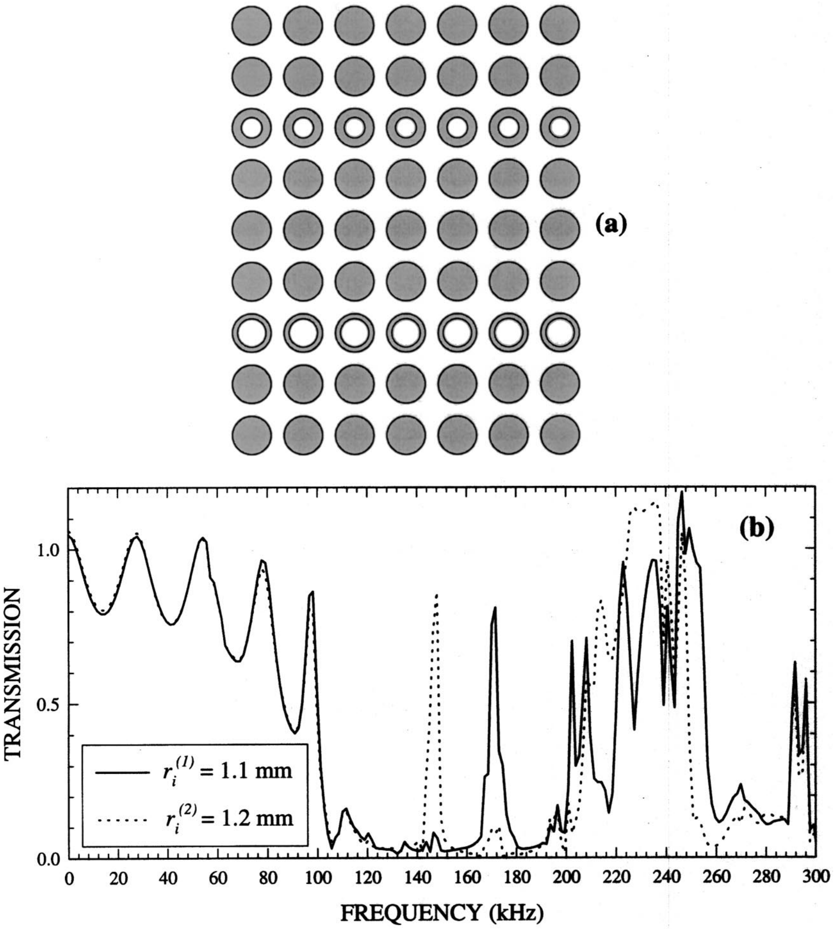

FIG. 4. (a) Two-dimensional cross section of two parallel NPB waveguides with tube inner radii of $r_i^{(1)}$ and $r_i^{(2)}$ equal to 1.10 and 1.20 mm, respectively. (b) Transmission spectra calculated at the exit of the two waveguides for the same input signal as that in Fig. 3(b).

in a narrow band around 148 kHz while the first transmits near 170 kHz. The weak transmissions at 148 kHz for the first guide and at 170 kHz for the second one are due to the fact that, for the sake of computational efficiency, we have separated the guides by only three rows of filled cylinders so some leakage between the guides occurs.

III. CONCLUSION

We have studied a class of two-dimensional phononic crystals with narrow pass bands inside their transmission gaps using the finite-difference time-domain method. The frequency of the NPB can be tuned by varying the inner radius of the tubular cylindrical inclusions. Simple qualitative rules of separation and superposition are applicable to predict the transmission spectrum of mixed PNB ABGs composed of nonuniform arrangements of cylinders with differing values of their inner radii. These rules form the basis for the design of elastic-acoustic waveguides that transmit at well defined tunable frequencies. We have also demonstrated the application of two of these tunable waveguides to the filtering and separation of specific frequencies from a broad band input signal.

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