Tunable filtering and demultiplexing in phononic crystals with hollow cylinders

Y. Pennec,[1,2] B. Djafari-Rouhani,[2] J. O. Vasseur,[2] A. Khelif,[3] and P. A. Deymier[4]
[1]*Laboratoire de Structure et Propriétés de l'Etat Solide, UMR CNRS 8008, Université de Lille1, 59655 Villeneuve d'Ascq Cedex, France*
[2]*Laboratoire de Dynamique et Structure des Matériaux Moléculaires, UMR CNRS 8024, Université de Lille1, 59655 Villeneuve d'Ascq Cedex, France*
[3]*Laboratoire de Physique et Métrologie des Oscillateurs, UPR CNRS 3203, Université de Franche-Comté, 32 Avenue de l'Observatoire, 25000 Besançon, France*
[4]*Department of Materials Science and Engineering, University of Arizona, Tucson, Arizona 85721, USA*
(Received 24 October 2003; published 29 April 2004)

Acoustic band gap (ABG) materials constituted of steel hollow cylinders immersed in water can exhibit a tunable narrow pass band (NPB) located inside their gap. We theoretically investigate, using the finite difference time domain (FDTD) method, the properties of waveguides composed of a row of hollow cylinders in a two-dimensional (2D) phononic crystal made of filled steel cylinders. These waveguides exhibit NPB's at frequencies slightly higher than their infinite periodic ABG counterpart. The frequency of the waveguide's NPB can be selected by adjusting the inner radius of the hollow cylinders or by changing the nature of the fluid that fills them. We show that a waveguide constituted of a row of hollow cylinders with different inner radii can transport waves at two different frequencies. By selectively filling the cylinders with water or mercury we have created an active device that permits the transmission of waves at one, both, or neither of these frequencies. Finally, we examine the multiplexing and demultiplexing capabilities of Y shaped waveguides constituted of hollow cylinders.

DOI: 10.1103/PhysRevE.69.046608 PACS number(s): 43.20.+g, 43.40.+s, 46.40.Cd, 63.20.−e

I. INTRODUCTION

Acoustic band gap (ABG) materials, also called phononic crystals, are inhomogeneous media constituted of two- or three-dimensional arrays of inclusions embedded in a matrix. The band structure of phononic crystals displays absolute gaps provided one imposes a large contrast between the elastic parameters and the mass density of the inclusions and of the matrix [1]. The existence of absolute band gaps was predicted theoretically [2–4] prior to being demonstrated experimentally in various phononic crystals constituted of solid components [5,6] or mixed solid/fluid components [7–9]. Recent studies have focused on (a) the acoustic behavior of linear and point defects [10] in phononic crystals, (b) wave bending and splitting [11], and (c) transmission through perfect [12,13] or defect-containing [13] waveguides. Waveguides in phononic crystals are produced by removing or replacing the cylindrical inclusions along one or several rows of the array. The insertion of defects, such as cavities, inside or at the edge of a waveguide [13] was shown to give rise to the filtering or to the rejection of selective frequencies in the guide's transmission spectrum. The ability to taylor the acoustic properties of phononic crystals and more specifically those of waveguides makes them particularly suitable for a wide range of applications from transducer technology to filtering and guidance of acoustic waves.

Recently, we have investigated a class of two-dimensional (2D) ABG materials that incorporate a tunable narrow pass band (NPB) in their gap [14]. These are constituted of a periodic repetition of hollow steel cylinders immersed in a liquid such as water. The frequency of the NPB can be tuned by modifying the inner radius of the hollow cylinders. In the present paper, we extend the concept of tunable NPB's from phononic crystals to waveguides. For this we consider the properties of a waveguide composed of a row of hollow cylinders in a 2D phononic crystal made of filled steel cylinders immersed in water. Since this phononic crystal can exhibit a broad band gap, one can investigate the filtering and multiplexing capabilities of the waveguide with NPB's. Using hollow cylinders with different inner radii, we are able to create several NPB's and transmit along the guide acoustic waves at two or more selected frequencies. Moreover, the transmitted signals can be actively tuned by changing the nature of the liquid that fill them, for instance, by replacing the water inside the hollow cylinders with a liquid such as mercury. Finally, we investigate the multiplexing and demultiplexing capabilities of Y shaped waveguides constituted of hollow cylinders.

The calculations performed in this work are based on the finite difference time domain (FDTD) method that has been proven to be an efficient method for obtaining both the transmission coefficient [5,15,16] and the dispersion curves [17] in phononic crystals.

The paper is organized as follows. The method of calculation is briefly presented in Sec. II. The FDTD results of the filtering and multiplexing properties of the new NPB waveguides are discussed in Sec. III. Some conclusions are given in Sec. IV.

II. METHOD OF CALCULATION AND GEOMETRICAL PARAMETERS

The FDTD method has been used with success extensively to study the propagation of electromagnetic waves through photonic band gap materials [18–20]. In recent years, the FDTD method has been extended to phononic crystals [5,15,16]. This method solves the elastic wave equation by discretizing time and space and by replacing deriva-

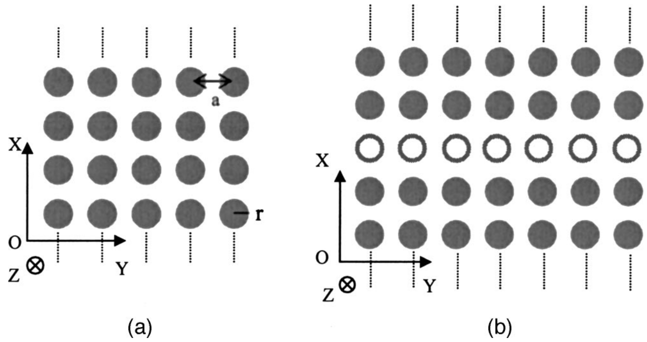

FIG. 1. (a) Two-dimensional cross section of perfect phononic crystal. We define a as the period of the phononic crystal and r as the radius of one cylinder. (b) The phononic crystal with a waveguide composed of hollow cylinders. The unit cell along the X direction contains five periods of the phononic crystal.

tives by finite differences. We apply the FDTD approach to calculate the transmission coefficient through finite thickness samples of phononic crystals. For this, we construct a sample in three parts along the direction of propagation Y. The central region contains the finite size inhomogeneous phononic composite. This region is sandwiched between two homogeneous media used for launching and probing the acoustic waves. Absorbing Mur boundary conditions [21] are imposed at the free ends of the homogeneous regions. The inhomogeneous region is constituted of cylindrical inclusions of infinite height along the Z direction, arranged periodically on a square lattice in the XY plane (Fig. 1). The lattice is finite along Y and comprises several periods ranging from five to thirteen. Along the X direction, the lattice is repeated periodically. When dealing with perfect phononic crystals, the lattice is limited to one period in the X direction. Because of periodic boundary conditions along X, a waveguide in a phononic crystal is also repeated periodically. To avoid interaction between a waveguide and its periodic images we employ, in that case, lattices with several periods (up to eight) along X (see Fig. 1).

A broad band wave packet is launched in the first homogeneous region. The incoming wave is a longitudinal pulse, uniform along the X direction and propagating with a Gaussian profile along the Y axis. The transmitted signal is recorded at the end of the second homogeneous region and integrated along the X direction. In all the calculations reported here, the period of the phononic crystal is taken to be $a=5$ mm. Accordingly, the space is discretized in both X and Y directions with a mesh interval of $(a/70)$. The equations of motion are solved with a time integration step of 4 ns. The number of time steps varies between 2^{19} and 2^{21} depending on the size of the system studied. The transmitted signal recorded as a function of time is Fourier transformed and normalized to the Fourier transform of a signal propagating through homogeneous water to yield the transmission coefficient. More details about the calculation can be found in Ref. [16].

The physical parameters characterizing the acoustic properties of the constitutive materials take the following values. The longitudinal and transverse speeds of sound in steel are

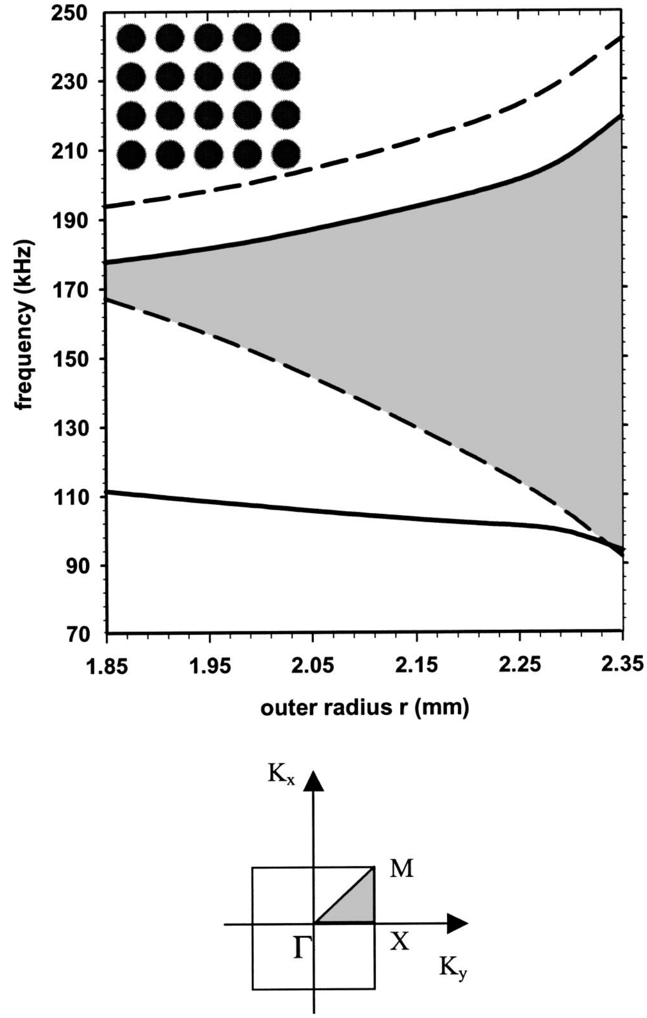

FIG. 2. Evolution of the band gap edges as a function of the radius of the cylinder in the case of a square lattice of filled steel cylinders embedded in water (see inset). The solid lines represent the upper and the lower edges of the gap for the ΓX direction of the irreducible Brillouin zone, the dashed lines represent the edges of the gap for the ΓM direction. The gray area corresponds to the absolute band gap. The irreducible Brillouin zone of the square lattice is also presented.

taken as 5825 and 3226 m/s. The longitudinal speed of sound in water is 1490 m/s. The mass densities of steel and water are 7.78 and 1.0 g/cm^3, respectively.

III. RESULTS

Before presenting the results of the calculations concerning the guiding and demultiplexing properties of waveguides, we briefly discuss the band structure of the perfect phononic crystal with either filled or hollow cylinders. This provides a basis for the development of waveguides in the subsequent sections.

A. Perfect phononic crystal with filled and hollow cylinders

We calculate the transmission coefficient through a perfect phononic crystal containing filled cylinders of steel,

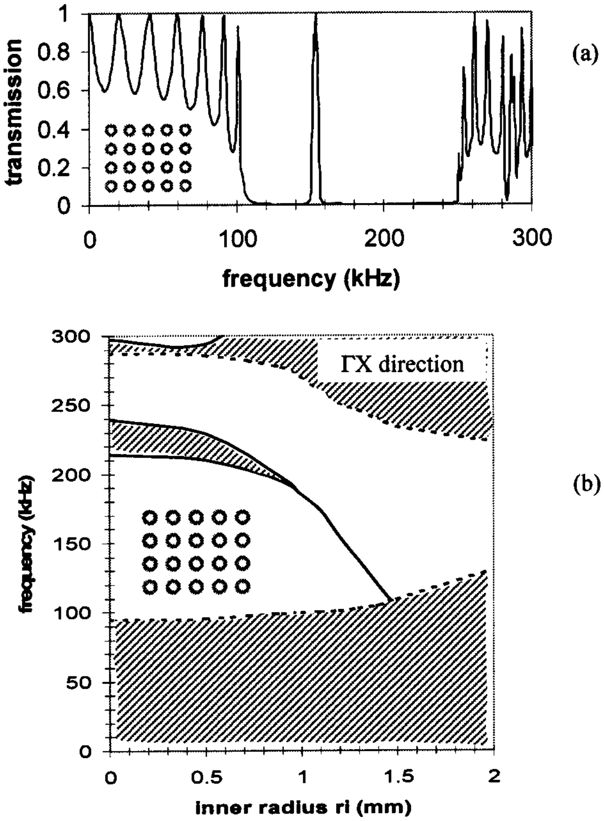

FIG. 3. (a) Transmission spectrum of the phononic crystal constituted of hollow steel cylinders with inner radius $r_i = 1.2$ mm (see inset). (b) in crosshatched: pass band frequencies as a function of the inner radius of hollow cylinders. The dashed lines represent the upper and the lower edges of the gap. Inside the gap, we can observe the evolution of a pass band (solid lines) that becomes a NPB for $r_i \geqslant 0.9$ mm.

along two high symmetry axes ΓX and ΓM of the Brillouin zone. The calculation is performed for several values of the radius r of the cylinders. In Fig. 2, we present the evolution of the band gap that separates the first two pass bands as a function of r. In order to obtain a wide band gap for both directions of propagation, an appropriate choice for the radius r will be $r = 2.3$ mm. This value will be kept fixed in the rest of the paper.

Next, we consider a perfect phononic crystal containing hollow cylinders of steel with variable inner radius r_i, outer radius $r = 2.3$ mm and period $a = 5$ mm. We have shown in a very recent paper [14] that such crystals can exhibit a tunable narrow pass band (TNPB) in their band gap. Figure 3(a) is an illustration of the TNPB obtained with an inner radius $r_i = 1.2$ mm. Considering the band structure [14], the NPB emerges from two almost flat dispersion curves along the ΓX direction of the Brillouin zone. The position of the narrow pass band as well as the top edge of the phononic gap are quite sensitive to the value of the inner radius r_i of the inclusions. To summarize the results of this calculation, we have reported in Fig. 3(b) the evolution of the passing bands (shaded area) and band gaps (white area) as a function of r_i. The width of the narrow band decreases with the radius r_i until the corresponding modes reach the lower edge of the

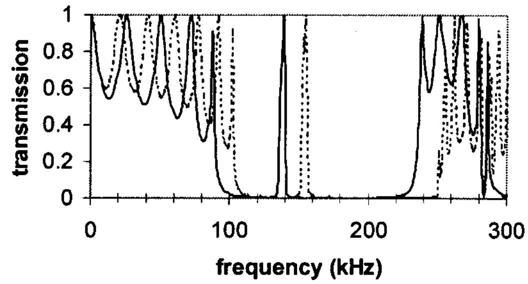

FIG. 4. Transmission spectra calculated for a phononic crystal composed of hollow cylinders with inner radius $r_i = 1.2$ mm containing mercury (solid line) or water (dashed line).

phononic gap. For a selective transmission through such a phononic crystal, an appropriate choice for r_i would be in the range of 0.9 to 1.4 mm.

With filtering and demultiplexing applications in mind, we now investigate the modification of the frequency of the narrow pass band when the hollow cylinders are filled with a fluid other than water. We chose mercury because of the large contrast between its physical parameters (mass density and compressibility) and those of water. The density of mercury is 13.6 g/cm^3 whereas the velocity of sound is equal to 1490 m/s. In Fig. 4, we report the transmission coefficient through the phononic crystal with hollow cylinders (inner radius 1.2 mm) filled with water or mercury. Upon changing the filling fluid from water to mercury, the frequency of the NPB shifts from 154 to 139 kHz.

Figure 5 summarizes the values of these NPB frequencies for other choices of the inner radius. The frequency shift between the water-filled cylinder NPB and mercury-filled cylinder NPB increases with increasing inner radius.

From the above discussion, one can conclude that the NPB of phononic crystals constituted of hollow cylinders can be tuned and may offer a mean for selective frequency transmission. The value of the frequency can be adjusted both by changing the inner radius of the cylinders or the nature of the fluid that fills them. In the latter case, the tuning of the frequency can be made either in a passive way or actively by injecting and flushing the fluids contained in the interior of the cylinders.

B. Guiding filtered acoustic waves

In this section, we are interested in using hollow cylinders to design waveguides in phononic crystals with NPB's. Here

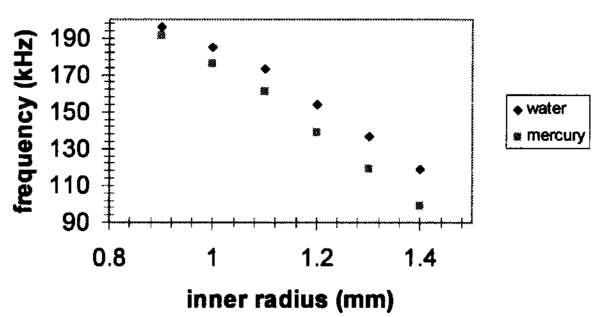

FIG. 5. Values of NPB center frequencies for phononic crystals of hollow cylinders with different inner radius containing mercury or water.

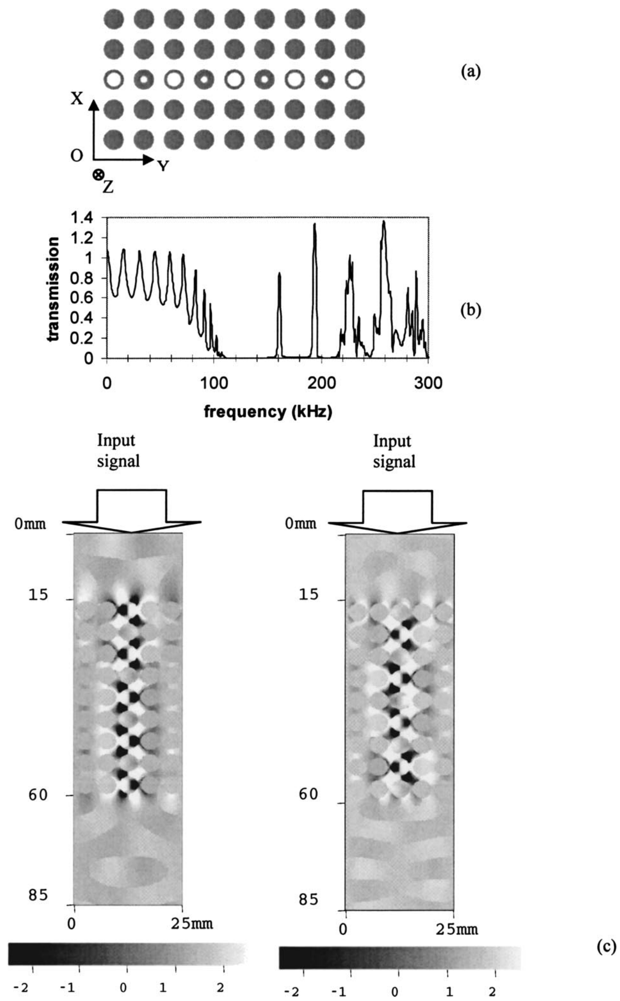

FIG. 6. (a) Two-dimensional cross section of the waveguide created inside a phononic crystal composed of filled cylinders in a water matrix. The NPB waveguide is composed of hollow cylinders with large (1.2 mm) and small (1.0 mm) inner radii in alternation. (b) Transmission spectrum of the mixed waveguide. (c) Representation of the displacement field for an alternated waveguide at the two frequencies of 161 kHz (left) and 194 kHz (right).

we study a 2D phononic crystal constituted of filled cylinders of steel immersed in water in which we created a linear waveguide [see Fig. 6(a)]. The waveguide is constituted of a row of hollow cylinders along the *Y* direction. The hollow cylinders constituting the guide can have either identical radii or different radii, subsequently called monoradius and heteroradii waveguides. In a first step, an heteroradii waveguide is constructed from alternating hollow cylinders with two different radii $r_i^{(1)}$ and $r_i^{(2)}$ [Fig. 6(a)]. For this system, the central region of the FDTD model contains five rows of nine cylinders arranged on a square lattice in the *Y* direction parallel to the wave propagation. The number of time integration step is increased from 2^{19} to 2^{20} because the system with a guide is longer than the perfect phononic crystals studied in Sec. III A. Because of periodic boundary conditions in the *X* direction, each guide is separated from its nearest periodic image by four rows of filled cylinders. This separation is sufficient to prevent significant leakage between the guides. We probe the transmission of the waveguide by launching the same broad band signal as introduced in Sec. II. However, the transmitted signal, recorded at the end of the second homogeneous region, is integrated only over the cross section of the waveguide instead of the whole period in the *X* direction as was the case for the perfect phononic crystals. One notes that, with this procedure, the maximum value of transmission can exceed the value of 1. The transmission through the heteroradii waveguide with the alternating large (1.2 mm) and small (1.0 mm) inner radii is displayed in Fig. 6(b). The spectrum exhibits a gap from 110 to 215 kHz. Inside the gap, we note the presence of two NPB modes occurring at 161 and 194 kHz. These frequencies differ significantly from those of the NPB in the perfect phononic crystal constituted of hollow cylinders (see Fig. 5). This means that the effect of confinement inside the waveguide is far from being negligible. Still, the narrowness of the pass bands is preserved.

In order to identify the two narrow pass bands we have calculated the transmission coefficient of two monoradius linear waveguides. For these systems, the central frequency of the NPB occurs at 161 and 194 kHz when the inner radius of the hollow cylinders equals 1.2 and 1.0 mm, respectively. Therefore, the heteroradii waveguide can transport both modes associated with each of the monoradii waveguides. We have represented in Fig. 6(c) maps of the displacement field for the two considered frequencies. As expected, we can observe a strong displacement field around the hollow cylinders with an inner radius of 1.2 mm when the transmission frequency is chosen to be 161 kHz. At the same time, the displacement field inside the hollow cylinders with an inner radius of 1.0 mm is very weak. In contrast, at the frequency of 194 kHz we observe a strong displacement field only around the hollow cylinders with $r_i = 1.0$ mm.

Similar conclusions hold when a mixed waveguide is constituted of hollow cylinders with the same inner radius but containing alternately two different fluids such as water and mercury. The transmission through such a waveguide is sketched in Fig. 7(a), where the central frequencies of the NPB occur at 161 and 143 kHz, for an inner radius $r_i = 1.2$ mm [Fig. 7(b)].

From the above observations, one could design an active device in which flushing and injecting specific fluids in the hollow cylinders would prevent transmission along the guide or permits the waveguide to transmit acoustic waves selectively at one or two desired frequencies corresponding to the NPB's. For instance, two frequencies are transmitted through the waveguide in the example of Fig. 7(a). Switching to a case in which all the cylinders are filled with mercury will suppress the transmission peak at 161 kHz and only the peak at 143 kHz can be observed at the waveguide exit. Now, switching to the case sketched in Fig. 7(c), where in different portions of the waveguide the cylinders are filled alternatively with water and mercury, will almost cancel the transmission in the whole range of the phononic crystal band gap [Fig. 7(d)]. Indeed, in this latter case, each segment of the waveguide allows the propagation of one NPB while prohibiting the propagation of the other NPB. The combination of the two effects will decrease the transmission of both NPB's.

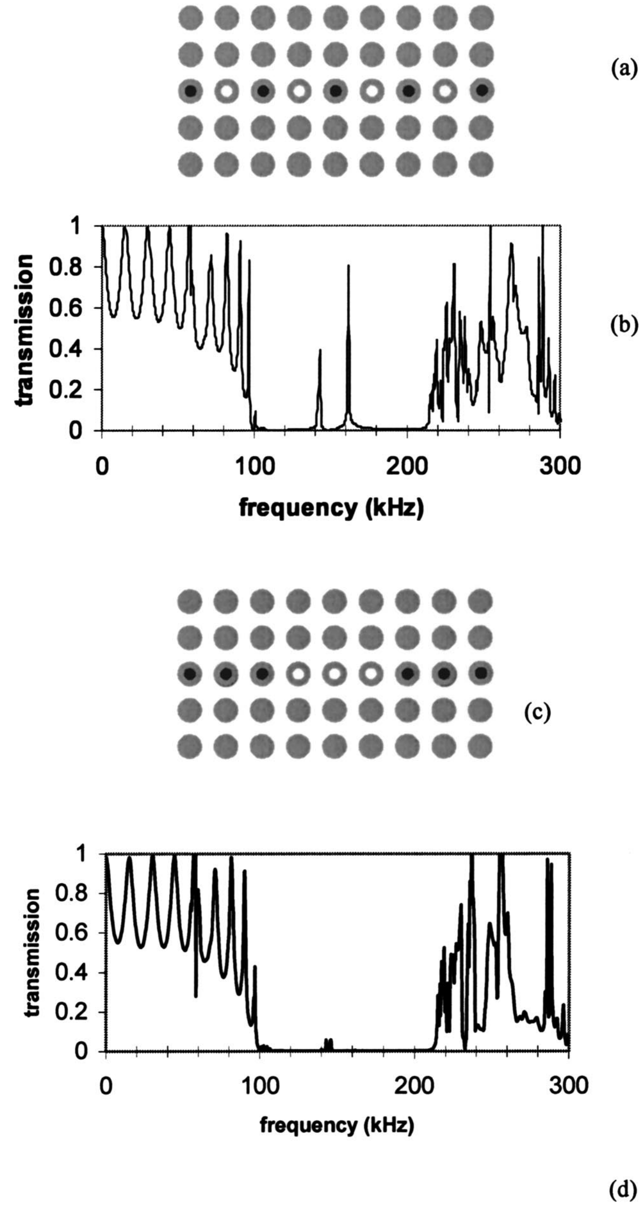

FIG. 7. (a) Two-dimensional cross section of the NPB waveguide composed of hollow steel tubes of same inner radius ($r_i = 1.2$ mm) but containing alternately mercury (black) and water (white). (b) FDTD transmission coefficient as a function of frequency for acoustic waves propagating in the mixed waveguide. (c) Same model of phononic crystal as in (a) but with a new arrangement of the hollow cylinders inside the mixed guide. (d) The transmission spectrum of the waveguide shown in (c).

C. Multiplexing filtered acoustic waves

In this section, we shall discuss the multiplexing and demultiplexing properties of Y-shaped waveguides constituted of hollows cylinders. Let us consider the geometry sketched in Fig. 8(a). The heteroradii waveguide with alternating radii $r_i = 1.2$ mm and $r_i = 1.0$ mm, will select two frequencies from a broad band input signal coming from the left (see Sec. III B). These frequencies correspond to the two NPB's of the heteroradii waveguide. This waveguide is then divided at its end into two branches. Each branch is constituted of

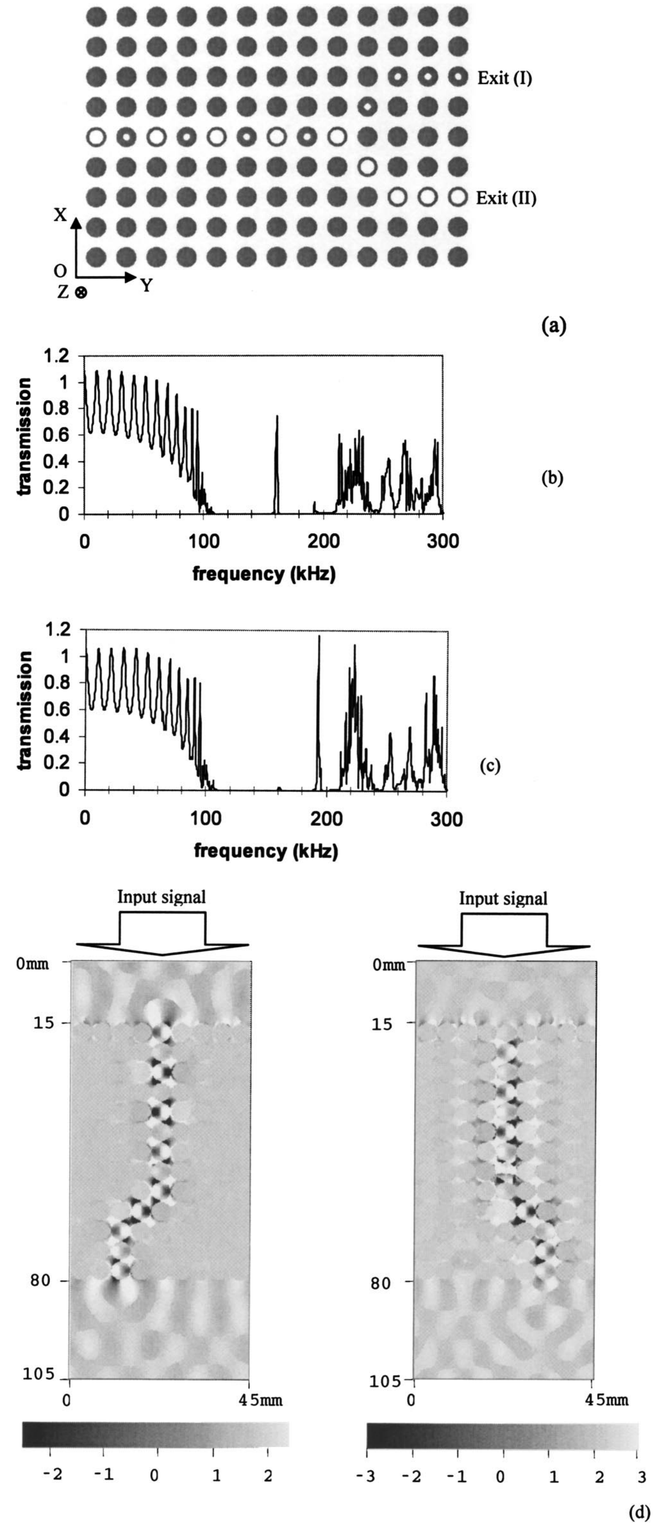

FIG. 8. (a) Schematic of the Y-shaped waveguide. The left part of the waveguide contains two types of cylinders with inner radii $r_i = 1.2$ and 1.0 mm, in alternation. Each branch of the Y contains one type of cylinder to permit the separation of the two NPB's. (b) Transmission spectrum when the detector is positioned at the exit (I) ($r_i = 1.2$ mm). (c) Transmission spectrum when the detector is positioned at the exit (II) ($r_i = 1.0$ mm). (d) Representation of the displacement field for a Y-shaped waveguide at the two frequencies of 161 kHz (left) and 194 kHz (right).

hollow cylinders designed for the propagation of waves with only one frequency corresponding to one specific NPB. We calculate the transmitted signals at the exit of each branch in the Y-shaped waveguide. In the FDTD calculation, the system contains nine periods a along the X direction in order to avoid interaction between the simulated guides and their periodic images. We chose a number of time integration steps equal to 2^{21} for this very large system. The transmission spectra calculated from the signals exiting the two branches of the Y-shaped waveguide are displayed in Figs. 8(b) and 8(c). These spectra clearly show that the superposed waves supported by the mixed waveguide have been separated and directed towards the two branches of the Y junction. The plot of the displacement field has been represented in Fig. 8(d) for the two considered frequencies 161 and 194 kHz.

It is also worth noticing that when the initial broad band signal is sent from the right of Fig. 8(a), each branch of the Y-shaped waveguide will select its own NPB. These two signals are then superimposed into the heteroradii waveguide. As a result, the transmitted spectrum at the left exit will contain two peaks corresponding to both NPB's. Finally, similar conclusions hold if the Y-shaped waveguide contains hollow cylinders of same inner radius filled with two different fluids.

IV. SUMMARY

In conclusion, we have investigated theoretically the propagation of acoustic waves through waveguides constituted of steel hollow cylinders incorporated in a phononic crystal constituted of solid cylinders of steel in water. Such systems could be used to transmit acoustic waves via very narrow pass band (NPB) inside a broad gap. The transmitted modes can be adjusted by selecting appropriately the inner radius of the constitutive hollow cylinders or the nature of the liquid filling the hollow tubes. We have studied a heteroradii waveguide constituted of hollow cylinders with alternating inner radii. This device permits the transmission of two superposed waves with different frequencies. Each one of these two frequencies is that of the NPB of a monoradius waveguide with the corresponding inner radius. NPB's can be similarly selected by creating a mixed waveguide composed of hollow cylinders with identical radii but filled with different fluids. By appropriately changing the fluid, one can design an active guiding device in which we can impose transmission at one of the frequencies or both frequencies of the NPB's in the transmission spectrum of the waveguide. Finally, we have discussed a model of multiplexer and demultiplexer based on a Y-shaped waveguide. This system can be utilized for separating or merging signals with different frequencies.

ACKNOWLEDGMENTS

The authors acknowledge Le Fond Européen de Développement Régional (FEDER) and Le Conseil Régional Nord–Pas de Calais for providing part of the computer facilities.

[1] For a review, see M. S. Kushwaha, Recent Res. Dev. Appl. Phys. **2**, 743 (1999).
[2] M. S. Kushwaha, P. Halevi, L. Dobrzynski, and B. Djafari-Rouhani, Phys. Rev. Lett. **71**, 2022 (1993); J. O. Vasseur, B. Djafari-Rouhani, L. Dobrzynski, M. S. Kushwaha, and P. Halevi, J. Phys.: Condens. Matter **6**, 8759 (1994).
[3] M. M. Sigalas and E. N. Economou, Solid State Commun. **86**, 141 (1993).
[4] M. S. Kushwaha, Appl. Phys. Lett. **70**, 3218 (1997).
[5] J. O. Vasseur, P. A. Deymier, G. Frantziskonis, G. Hong, B. Djafari Rouhani, and L. Dobrzynski, J. Phys.: Condens. Matter **10**, 6051 (1998); J. O. Vasseur, P. A. Deymier, B. Chenni, B. Djafari Rouhani, L. Dobrzynski, and D. Prevost, Phys. Rev. Lett. **86**, 3012 (2001).
[6] R. E. Vines, J. P. Wolfe, and A. V. Every, Phys. Rev. B **60**, 11 871 (1999).
[7] F. R. Montero de Espinosa, E. Jimenez, and M. Torres, Phys. Rev. Lett. **80**, 1208 (1998).
[8] W. M. Robertson and J. F. Rudy III, J. Acoust. Soc. Am. **104**, 694 (1998).
[9] J. V. Sanchez-Pérez, D. Caballero, R. Martinez-Sala, C. Rubio, J. Sanchez-Dehesa, F. Meseguer, J. Llinares, and F. Galvez, Phys. Rev. Lett. **80**, 5325 (1998); D. Caballero, J. Sanchez-Dehesa, J. Rubio, R. Martinez-Sala, J. V. Sanchez-Perez, F. Meseguer, and J. Llinares, Phys. Rev. E **60**, R6316 (1999).
[10] M. Torres, F. R. Montero de Espinosa, D. Garcia Pablos, and N. Garcia, Phys. Rev. Lett. **82**, 3054 (1999).
[11] M. Torres, F. R. Montero de Espinosa, and J. L. Aragon, Phys. Rev. Lett. **86**, 4282 (2001).
[12] M. Kafesaki, M. M. Sigalas, and N. Garcia, Phys. Rev. Lett. **85**, 4044 (2000); Physica B **296**, 190 (2001).
[13] A. Khelif, B. Djafari-Rouhani, J. O. Vasseur, and P. A. Deymier, Phys. Rev. B **68**, 024302 (2003).
[14] A. Khelif, P. A. Deymier, B. Djafari-Rouhani, J. O. Vasseur, and L. Dobrzynski, J. Appl. Phys. **94**, 1308 (2003).
[15] M. M. Sigalas and N. Garcia, J. Appl. Phys. **87**, 3122 (2000).
[16] See, for instance, Ph. Lambin, A. Khelif, J. O. Vasseur, L. Dobrzynski, and B. Djafari-Rouhani, Phys. Rev. E **63**, 066605 (2001), and references therein.
[17] Y. Tanaka and S. Tamura, Phys. Rev. B **58**, 7958 (1998); *ibid.* **60**, 13294 (1999).
[18] A. Taflove, *The Finite Difference Time Domain Method* (Artech, Boston, 1998).
[19] C. T. Chan, Q. L. Yu, and K. M. Ho, Phys. Rev. B **62**, 7387 (2000).
[20] S. Fan, P. R. Villeneuve, and J. D. Joannopoulos, Phys. Rev. B **54**, 11 245 (1996).
[21] G. Mur, IEEE Trans. Electromagn. Compat. **23**, 377 (1981).