J. Phys.: Condens. Matter **10** (1998) 6051–6064. Printed in the UK PII: S0953-8984(98)93210-6

Experimental evidence for the existence of absolute acoustic band gaps in two-dimensional periodic composite media

J O Vasseur†, P A Deymier‡, G Frantziskonis§, G Hong§, B Djafari-Rouhani† and L Dobrzynski†

† Equipe de Dynamique des Interfaces, Laboratoire de Dynamique et Structures des Matériaux Moléculaires, URA CNRS No 801, UFR de Physique, Université de Lille I, 59655 Villeneuve d'Ascq Cédex, France

‡ Department of Materials Science and Engineering, University of Arizona, Tucson, AZ 85721, USA

§ Department of Civil Engineering, University of Arizona, Tucson, AZ 85721, USA

Received 9 April 1998

Abstract. Transmission of acoustic waves in two-dimensional binary solid/solid composite media composed of arrays of Duralumin cylindrical inclusions embedded in an epoxy resin matrix is studied. The experimental transmission spectrum and theoretical band structure of two periodic arrays of cylinders organized on a square lattice and on a centred rectangular network are reported. Absolute gaps extending throughout the first two-dimensional Brillouin zone are predicted. The measured transmission is observed to drop to noise level throughout frequency intervals in reasonable agreement with the calculated forbidden frequency bands.

1. Introduction

During the last few years, further to Yablonovitch's structural proposal [1], the propagation of electromagnetic waves in artificial periodic structures of dielectric materials, known as 'photonic crystals', has received a great deal of attention. Of particular interest is the existence of forbidden frequency bands in which electromagnetic modes, spontaneous emission and zero-point fluctuations are all absent [2].

This search for band gaps in 'photonic crystals' and the mathematical analogy between electromagnetic waves and vibrations has spurred a 'renewed interest' in the propagation of elastic waves in the so-called 'phononic crystals'. The propagation of elastic waves in periodic or random composite materials is an old topic in condensed matter physics and/or acoustics [3, 4]. The present research activities focus on the design of band gaps in the acoustic spectrum of composite media. Acoustic gaps being frequency domains in which propagation of sound and phonons are forbidden, one can imagine for these phononic crystals numerous engineering applications such as frequency filters, vibrationless environments for high-precision mechanical systems or the design of new transducers. Recent theoretical results have shown the existence of gaps in the acoustic band structures of 3D and 2D phononic crystals of various compositions. More precisely large acoustic gaps were obtained for cubic lattices of elastic spherical inclusions surrounded by a host matrix, the constituent materials being either both solids [5, 6] or both fluids [7, 8]. 2D phononic crystals composed of solid (or fluid) cylindrical inclusions periodically placed in an elastic solid (or fluid) background may also present large acoustic gaps, this for inclusions situated on square

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[6, 9–12], triangular [6, 13–15] or boron-nitride-like lattices [16]. Most of these recent studies identified absolute band gaps in the vibration spectrum of the composites, that is, gaps extending throughout the Brillouin zone irrespective of the direction of propagation. For two-dimensional composite materials, these absolute band gaps were obtained with the Bloch wave vector perpendicular to the cylindrical inclusions. In 2D and 3D composites, the contrast in elastic properties and densities between the constituents, and the composition of the inhomogeneous artificial material, are emerging as critical parameters in determining not only the existence of acoustic gaps but also their width.

On the other hand, experimental studies of the propagation of elastic waves have been done by several authors. Acoustic properties of fibrous materials composed of W wires embedded in an Al matrix [17] and of particulate (e.g. 3D) glass/epoxy or steel/PMMA composites [18] were investigated. In these studies, jumps in the wave phase velocity measured as a function of frequency were assigned to stop-bands in the dispersion curve of the inhomogeneous material. In both studies, the volume fraction of inclusions was lower than 25% and, in the light of recent results on 2D and 3D composite systems [5, 6, 9–12], one may think that these stop-bands are local gaps and do not extend over the whole Brillouin zone. Martinez-Sala *et al* [19] have experimentally determined the sound attenuation spectrum in a sculpture made of a 2D array of hollow stainless steel cylinders in air. These authors ascribed the existence of a peak in this spectrum to the formation of a phononic gap in this sculpture. One can notice that Sigalas and Economou [20] and Kushwaha [21] have shown that this sculpture exhibits only pseudogaps, not absolute gaps. In [20] and [21], the authors have assumed the steel inclusions as infinitely rigid. Under this restrictive hypothesis, the array of steel cylinders in air can be treated as a fluid/fluid composite. This assumption is not valid for common binary solid/fluid composites and reliable theoretical model for the propagation of acoustic waves in this kind of composite material is not yet available. In a previous paper [22], we have shown the existence of strong acoustic absorptions in the P-wave experimental transmission spectrum of a 2D composite medium. This composite was constituted of parallel Duralumin (Al 95%–Cu 4%–Mg 1% alloy) cylinders arranged periodically on a square lattice and embedded in a polyvinyl chloride (PVC) matrix, the volume fraction of inclusions being 12.6%. These absorptions were assigned to local gaps in the theoretically calculated elastic band structure. More recently, Montero de Espinosa *et al* [23] have experimentally observed a full band gap for the longitudinal waves in a 2D fluid/solid composite material. This sample is constituted of mercury cylinders arranged on a square array and embedded in an aluminum matrix with a filling fraction of 40%. Unfortunately the theoretical prediction of this full band gap is lacking in this paper.

Although there is strong evidence supporting the fact that composite materials may exhibit stop bands in their acoustic spectrum, the design of composite acoustic systems with tailored gaps requires the development of reliable models with predictive capability. This paper presents a comparative study between theoretically predicted gaps within the well known plane wave method and experimentally measured gaps of finite size samples.

The goals of the present work are twofold. The first one consists of a search and measurement of absolute acoustic band gaps in 2D solid/solid composite media; in a second step a theoretical model commonly used in the computation of acoustic band structures of such composites is validated by direct comparison with measured results. For this, we have manufactured two composite samples constituted of Duralumin cylinders inserted in an epoxy resin matrix. The array of inclusions is square in one case and centred rectangular in the other. The centred rectangular geometry was chosen because of its reduced symmetry. We have measured experimentally the acoustic wave transmission spectrum of both samples. Both samples exhibit sharp drops in transmitted intensity in some frequency ranges serving as strong evidence for the existence of absolute band gaps in 2D composite media. Comparison of the experimental results with the calculated band structures of these same periodic 2D systems shows good qualitative agreement. In addition, reasonable agreement between the interval of frequencies over which transmission is minimal and theoretically predicted absolute band gaps is achieved. However, there exist some points of disaccord in the comparison between the calculated and the experimental gaps which are, in particular, imputed to the finite nature of our samples, especially for the centred rectangular array.

This paper is organized as follows. In section 2, we present the samples, their preparation process and the experimental setup as well as the experimental results for their acoustic transmission spectra. The theoretical method for the calculation of the acoustic band structures of 2D binary solid/solid composite media is summarized in section 3 followed by the numerical results for the calculated band structures. The experimental and theoretical results are then compared and discussed in section 4. Some conclusions concerning the agreements and points of contention between experiments and theory are also drawn in that section along with some future perspectives.

2. Experimental method

2.1. Composite systems

The two samples of binary composite materials are constituted of arrays of 25 and 23 parallel cylinders of Duralumin arranged on a square and a centred rectangular lattices, respectively. The cylinders are embedded in an epoxy resin matrix. The choice of these usual materials is based on the strong contrast in their elastic constants and densities [5–16]. These data are listed in table 1. The metallic cylinders have a diameter $d = 16$ mm. The periodicity of the square lattice is $a = 20$ mm. The centred rectangular lattice has periodicities of $a = 20$ mm and $b = 2a = 40$ mm. The filling fraction of metal to resin, defined as the ratio between the cross-sectional area of one rod and the surface of one unit cell, is given in both cases by $f = \pi d^2/4a^2$ and is equal to 0.503. The physical dimensions of the samples are 10 cm $\times$ 10 cm $\times$ 10 cm. We have illustrated in figures 1(a) and 1(b) the two-dimensional cross sections of these specimens.

Table 1. The densities and elastic constants of Duralumin and epoxy resin [24]. C_l and C_t represent the longitudinal and the transverse velocities of sound, respectively.

	ρ (g cm^{-3})	C_l (m s^{-1})	C_t (m s^{-1})	$C_{11} = \rho C_l^2$ (10^{10} N m^{-2})	$C_{44} = \rho C_t^2$ (10^{10} N m^{-2})
Duralumin	2.799	6342	3095	11.26	2.681
Epoxy	1.142	2569	1139	0.754	0.148

2.2. Sample preparation

Two cubic moulds, 10.5 cm $\times$ 10.5 cm $\times$ 10.5 cm, were fabricated by assembling five square Duralumin plates. 16 mm diameter $\times$ 2 mm deep bores, arranged on square (or centred rectangular) arrays, were drilled on two opposite sides of the mould to maintain the metal cylinders. Clean Duralumin cylinders were then placed and fixed between these

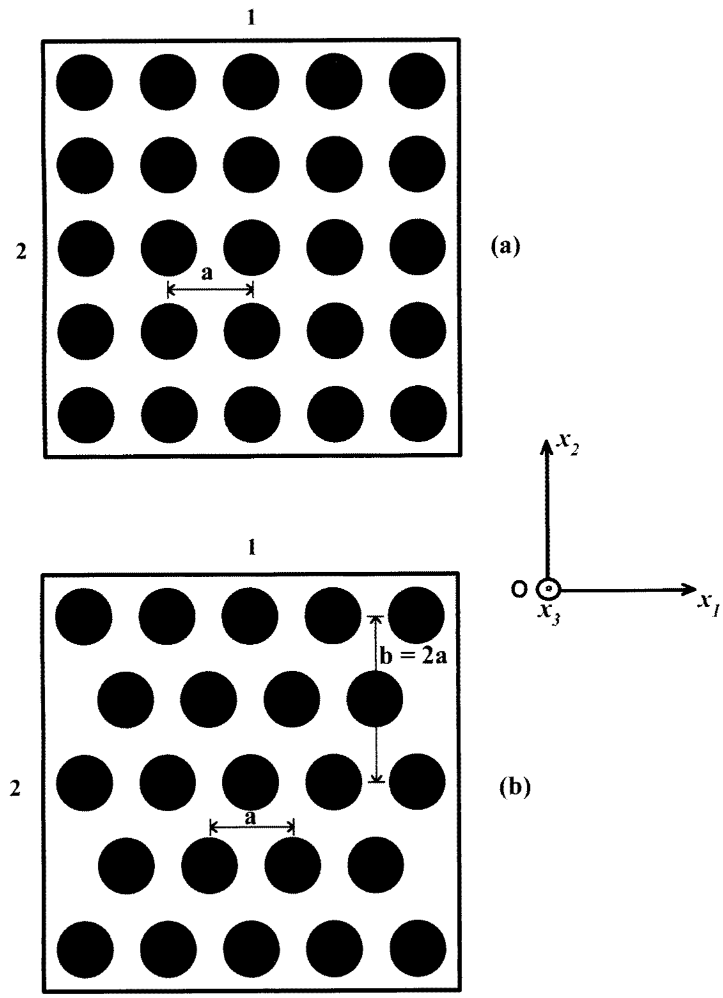

Figure 1. Two-dimensional cross section of the square (a) and centred rectangular (b) arrays of Duralumin cylinders embedded in an epoxy matrix. The probed faces of the specimen are labelled (1) and (2). The cylinders are parallel to the x_3 axis of the Cartesian coordinate system $(Ox_2x_2x_3)$.

two plates. The liquid epoxy resin [24] was poured in the mould and degassed during a long period of time to insure that most of the trapped air was evacuated. After hardening, the composite material was removed from the mould and the faces of the specimen were pumiced and polished in order to obtain surfaces as smooth as possible.

2.3. Experimental setup

The ultrasonic emission source used in the experiment is a Panametrics delta broad-band 500 kHz P-transducer with pulser/receiver model 500PR. The measurement of the signals is performed with a Tektronix TDS 540 oscilloscope equipped with a TD100 Data Manager. The transducers are cylindrical with a diameter of 3.175 cm (1.25 inch). The transducers are centred on the faces of the composite specimen and the nearly parallel signal is perpendicular to the Duralumin cylinders. Emission source produces compression waves (P-waves) and the receiving transducer detects only the longitudinal component of the transmitted wave. The measured transmitted signal is Fourier transformed to produce a power spectrum. Measurements have been performed on the faces (1) and (2) of the two periodic composite media as illustrated in figure 1. Because of the fact that the size of the transducer diameter is only approximately 50% larger than the periodicity, a, of the arrays of cylinders, measurements were conducted at several positions on the faces of the sample and were averaged to produce a compounded transmission spectrum.

The compounded power spectrum for the square array probed on face (1) is reported in figure 2(a). This compounded spectrum is the average of two spectra. These two spectra were obtained by centring the transducer on a row of cylinders or between two rows of cylinders, respectively. In light of the symmetry of the square lattice, we have verified that identical measurements are obtained perpendicular to face (2) of this sample. The spectra for faces (1) and (2) of the centred rectangular composite are presented in figures 2(b) and 2(c), respectively. The transmission spectrum of figure 2(b) is an average of a measurement along a row of two cylinders and a measurement along a row of three cylinders. The compounded spectrum of figure 2(c) is again the average of two spectra generated by placing the transducer along the central row of cylinders and between adjacent rows of cylinders. The transmitted spectra have been measured under the same experimental conditions, in particular the same gain, such that a comparison between the transmitted intensities can be made. The transmission spectrum of the square array exhibits a well defined drop in intensity between 50 and 130 kHz. This region of the spectrum is composed of an interval of frequencies 55–85 kHz where only noise level intensity is measured, followed by some transmitted intensity between 85 and 115 kHz. Between 115 and 125 kHz, the material does not transmit significantly. Again in this region the transmitted intensity is within the noise level of the equipment. A sharp rise in transmission occurs after 125 kHz. The two regions with noise level intensity are transmission gaps. Because of its geometry, the sample with a CR array of cylinders is strongly anisotropic. The transmission spectra measured perpendicular to faces (1) and (2) clearly reflect this anisotropy. One observes in figures 2(b) and 2(c) transmission peaks of lower height than in figure 2(a). This indicates a larger attenuation than in the case of the sample with a square array. The identification of a transmission gap for this sample is then more difficult. However, in spectrum 2(b) the transmitted intensity drops significantly in the interval of frequency 50–130 kHz and only noise level intensity is measured between 55 and 85 kHz. The attenuation of the ultrasonic signal is much more important perpendicularly to face (2) of the sample. In spectrum 2(c), low transmission is observed in the interval of frequency 50–100 kHz. From figures 2(b) and 2(c) the noise level frequency intervals intersect in the range of frequency 55–85 kHz which can then be identified as one stop-band for this sample.

3. Theoretical results

3.1. Model and method of calculation

We calculate elastic band structures for solid/solid periodic 2D binary composite systems using a method developed by Kushwaha *et al* [10–12]. These periodic systems are modelled as arrays of infinite cylinders of circular cross section made of isotropic materials, A, embedded in an isotropic matrix B. Matrix and inclusions are described within the context of linear elasticity. The elastic cylinders, of diameter d, are assumed to be parallel to the x_3

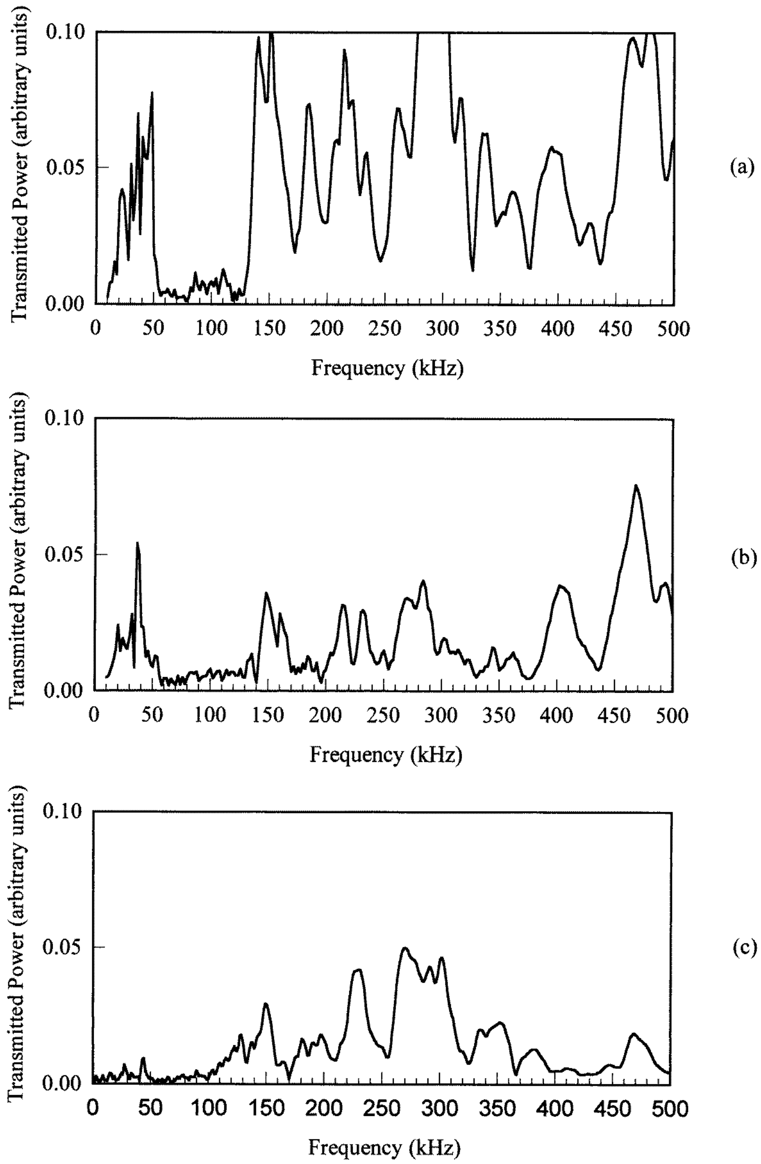

Figure 2. Transmission power spectrum measured perpendicular to (a) the face (1) for the square array, (b) the face (1) for the centred rectangular array and (c) the face (2) for the centred rectangular array.

axis of the Cartesian coordinate system $(\mathrm{O}x_1x_2x_3)$ ($\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ are unit vectors along the x_1, x_2, x_3 axes, respectively). The array is then considered infinite in the three directions x_1, x_2 and x_3. The intersections of the cylinder axes with the $(x_1\mathrm{O}x_2)$ transverse plane form a two-dimensional periodic array of specific geometry. In this work, we consider two kinds of infinite periodic array, namely a square lattice and a centred rectangular lattice with lattice parameters identical to those of figure 1. We investigate the propagation of elastic waves in the $(x_1\mathrm{O}x_2)$ transverse plane, that is we consider a Bloch wave vector perpendicular to the cylinders.

In the $(Ox_1x_2x_3)$ Cartesian coordinate system, the primitive lattice vectors for the square and centred rectangular structures can be written respectively as

$$\begin{cases} \boldsymbol{a}_1 = (a, 0) \\ \boldsymbol{a}_2 = (0, a) \end{cases} \tag{1a}$$

$$\begin{cases} \boldsymbol{a}_1 = \left(\frac{a}{2}, a\right) \\ \boldsymbol{a}_2 = \left(-\frac{a}{2}, a\right). \end{cases} \tag{1b}$$

The origin O is chosen on one site of the array.

The primitive vectors of the reciprocal lattices for the square (S) and the centred rectangular (CR) arrays are respectively given by

$$\begin{cases} \boldsymbol{b}_1 = \frac{2\pi}{a}(1, 0) \\ \boldsymbol{b}_2 = \frac{2\pi}{a}(0, 1) \end{cases} \tag{2a}$$

$$\begin{cases} \boldsymbol{b}_1 = \frac{2\pi}{a}\left(1, \frac{1}{2}\right) \\ \boldsymbol{b}_2 = \frac{2\pi}{a}\left(-1, \frac{1}{2}\right). \end{cases} \tag{2b}$$

The two-dimensional reciprocal lattice vectors $\boldsymbol{G}$ for the square array are

$$\boldsymbol{G} = h_1\boldsymbol{b}_1 + h_2\boldsymbol{b}_2 = \frac{2\pi}{a}[h_1\boldsymbol{e}_1 + h_2\boldsymbol{e}_2]. \tag{3a}$$

In the case of the centred rectangular lattice, they take the form

$$\boldsymbol{G} = h_1\boldsymbol{b}_1 + h_2\boldsymbol{b}_2 = \frac{2\pi}{a}[(h_1 - h_2)\boldsymbol{e}_1 + \tfrac{1}{2}(h_1 + h_2)\boldsymbol{e}_2]. \tag{3b}$$

In relations (3a) and (3b), h_1 and h_2 are two integers.

The method for the calculation of the band structure is the well known plane wave method where the densities and the elastic constants of the isotropic constituent materials, which are position dependent in the composite system, are developed in 2D Fourier series in the reciprocal space. We summarize here this method for 2D systems. The mass density and the elastic constants are ρ_A, C_{11A} and C_{44A} inside the cylinders A and ρ_B, C_{11B} and C_{44B} in the background (i.e. matrix) B. These physical characteristics in the composite system, denoted ζ in a general way, are space dependent with respect to the position vector $\boldsymbol{r} = (x_1, x_2)$ in the transverse plane, i.e. $\zeta(\boldsymbol{r}) = \zeta_A$ in the cylinder A and $\zeta(\boldsymbol{r}) = \zeta_B$ in the matrix B.

In the 2D binary composite material, in the absence of external force, the equations of motions are

$$\rho(\boldsymbol{r})\frac{\partial^2 u_i}{\partial t^2} = \boldsymbol{\nabla}\cdot[C_{44}(\boldsymbol{r})\boldsymbol{\nabla}u_i] + \boldsymbol{\nabla}\left[C_{44}(\boldsymbol{r})\frac{\partial \boldsymbol{u}}{\partial x_i}\right] + \frac{\partial}{\partial x_i}[(C_{11}(\boldsymbol{r}) - 2C_{44}(\boldsymbol{r}))\boldsymbol{\nabla}\cdot\boldsymbol{u}] \tag{4}$$

where $\boldsymbol{u}(\boldsymbol{r}, t)$ is the position and harmonic time dependent displacement vector of components u_i $(i = 1, 2, 3)$ in the Cartesian coordinate system $(Ox_1x_2x_3)$. If we limit the wave propagation to the (x_1Ox_2) transverse plane, one can introduce a 2D wave vector

$\boldsymbol{K}(K_1, K_2)$ (which means $K_3 = 0$) and the harmonic displacement vector $\boldsymbol{u}$ is independent of the x_3 coordinate. Then, equation (4) can be separated into the following two equations:

$$\rho(\boldsymbol{r})\frac{\partial^2 u_i}{\partial t^2} = \boldsymbol{\nabla}_T \cdot [C_{44}(\boldsymbol{r})\boldsymbol{\nabla}_T u_i] + \boldsymbol{\nabla}_T \cdot \left[C_{44}(\boldsymbol{r})\frac{\partial \boldsymbol{u}_T}{\partial x_i}\right] + \frac{\partial}{\partial x_i}[C_{11}(\boldsymbol{r}) - 2C_{44}(\boldsymbol{r}))\boldsymbol{\nabla}_T \cdot \boldsymbol{u}_T] \tag{5}$$

($i = 1$ or 2) with $\boldsymbol{u}_T = u_1\boldsymbol{e}_1 + u_2\boldsymbol{e}_2$ and $\boldsymbol{\nabla}_T = \boldsymbol{e}_1\partial/\partial x_1 + \boldsymbol{e}_2\partial/\partial x_2$ and

$$\rho(\boldsymbol{r})\frac{\partial^2 u_3}{\partial t^2} = \boldsymbol{\nabla} \cdot [C_{44}(\boldsymbol{r})\boldsymbol{\nabla} u_3]. \tag{6}$$

Equation (6) corresponds to pure transverse modes of vibrations ($u_3\boldsymbol{e}_3 \perp \boldsymbol{K}$) called Z modes. Equation (5) describes modes of vibrations for which $\boldsymbol{u}_T$ and $\boldsymbol{K}$ are coplanar vectors and are denoted as XY modes [10].

In our calculations of elastic band structure of 2D binary composite systems, equations (5) and (6) form the basic equations describing the propagation of acoustic waves. Taking advantage of the 2D periodicity in the $(x_1\mathrm{O}x_2)$ plane, the quantities $\rho(\boldsymbol{r})$, $C_{11}(\boldsymbol{r})$, $C_{44}(\boldsymbol{r})$ for composite inhomogeneous media are developed in Fourier series in the form

$$\zeta(\boldsymbol{r}) = \sum_{\boldsymbol{G}} \zeta(\boldsymbol{G})\, \mathrm{e}^{\mathrm{i}\boldsymbol{G}\cdot\boldsymbol{r}}. \tag{7}$$

The Fourier coefficients in equation (7) are given as

$$\zeta(\boldsymbol{G}) = \zeta_B\delta_{\boldsymbol{G}0} + (\zeta_A - \zeta_B)F(\boldsymbol{G}) \tag{8}$$

where δ is the Kronecker symbol. In equation (8), $F(\boldsymbol{G})$ stands for the structure factor [10–12] of the cylinder A defined as $F(\boldsymbol{G}) = 2fJ_1(GR)/GR$ where $J_1(x)$ is the Bessel function of the first kind of order one.

After some algebra, equations (5) and (6) become standard eigenvalue equations for which the size of the involved matrices depends on the number of $\boldsymbol{G}$ vectors taken into account in the truncated Fourier series.

3.2. Numerical results for the square and centred rectangular lattices

In this sub-section, we present band structures calculated for infinite periodic square (S) and centred rectangular (CR) lattices of Duralumin cylinders in an epoxy matrix. We focus on XY modes of vibration and we solve the Fourier transform of equation (5) numerically. In these calculations, 441 and 453 shortest $\boldsymbol{G}$ vectors are taken into account for the S and CR systems, respectively. These numbers of $\boldsymbol{G}$ vectors ensure sufficient convergence of the calculated eigenvalues and offer a good compromise between accuracy and computing time. The elastic parameters used in these calculations are those given in section 2.1.

Figures 3 and 4 show the XY mode band structure for the infinite S and CR arrays of Duralumin cylinders in epoxy resin. These dispersion curves have been plotted in the principal symmetry directions of the Brillouin zone (see insets in figures 3 and 4). The plots are given in terms of the frequency ν in kHz versus the reduced Bloch wave vector $\boldsymbol{k} = \boldsymbol{K}a/2\pi$ for ν in the range $0 < \nu < 200$ kHz. These two figures are quite similar. For the square array, a large gap opens up approximately between the maximum of the third band (point X) and the minimum of the fourth band located on the middle of segment ΓM. This gap extends from $\nu = 57$ kHz to $\nu = 88$ kHz. For the centred rectangular array, a gap appears between the maximum ($\nu = 59$ kHz) of the third band located on the middle of segment XJ and the minimum of the fourth band at point Γ ($\nu = 87$ kHz). The width

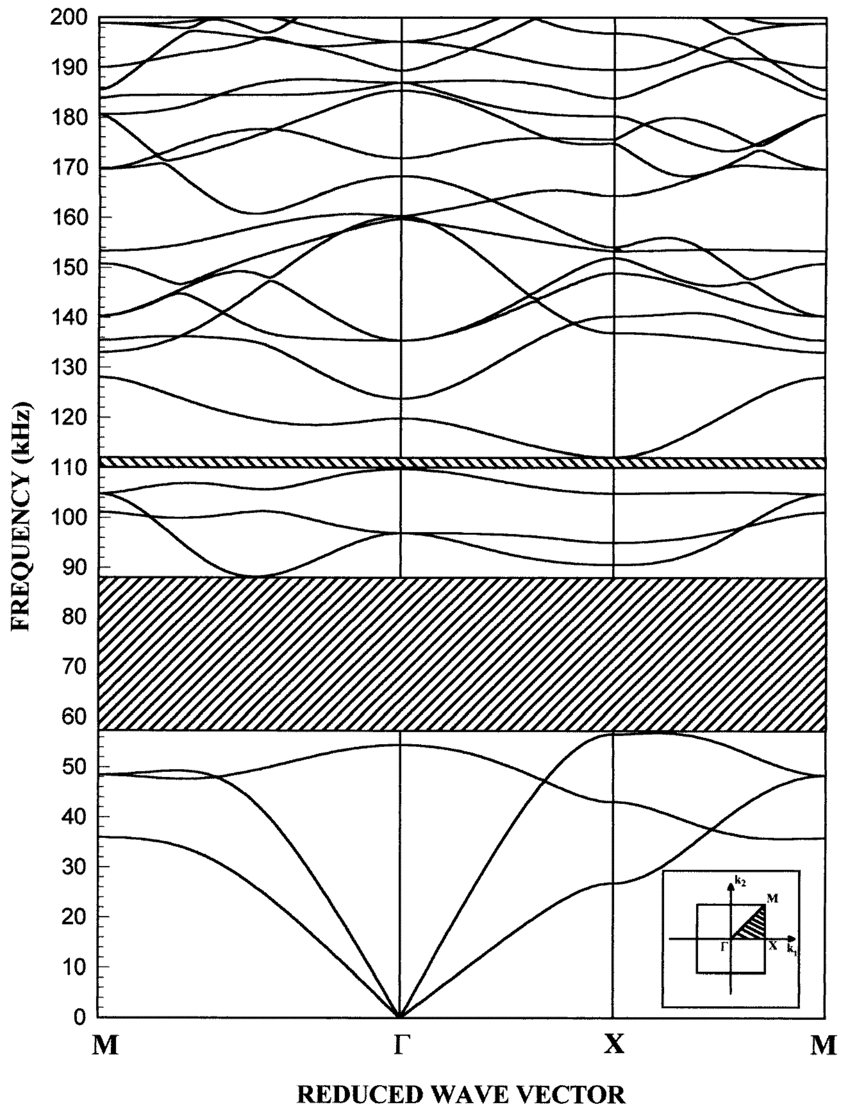

Figure 3. Band structure for the two-dimensional XY modes of vibration in the periodic square array of Duralumin cylinders in an epoxy resin matrix for $f = 0.503$. The inset shows the irreducible Brillouin zone. The reduced wave vector is defined as $\boldsymbol{K}a/2\pi$ where $\boldsymbol{K}$ is a two-dimensional wave vector. The points Γ, X and M in the 2D Brillouin zone have reduced components $(0, 0)$, $(1/2, 0)$ and $(1/2, 1/2)$. Absolute band gaps are represented as hatched areas.

of the first gap for both arrays extend over nearly the same frequencies. We note that the centred rectangular array is similar to a distorted triangular lattice [13, 16]. With the same filling fraction ($f \approx 50\%$), the computed XY band structure of an infinite triangular array of Dural cylinders in epoxy presents a large gap of similar width. In figures 3 and 4, there exists also a narrow gap around approximately $\nu = 110$ kHz. In both cases, we have also calculated the densities of states of the XY modes, scanning the surface of the two-

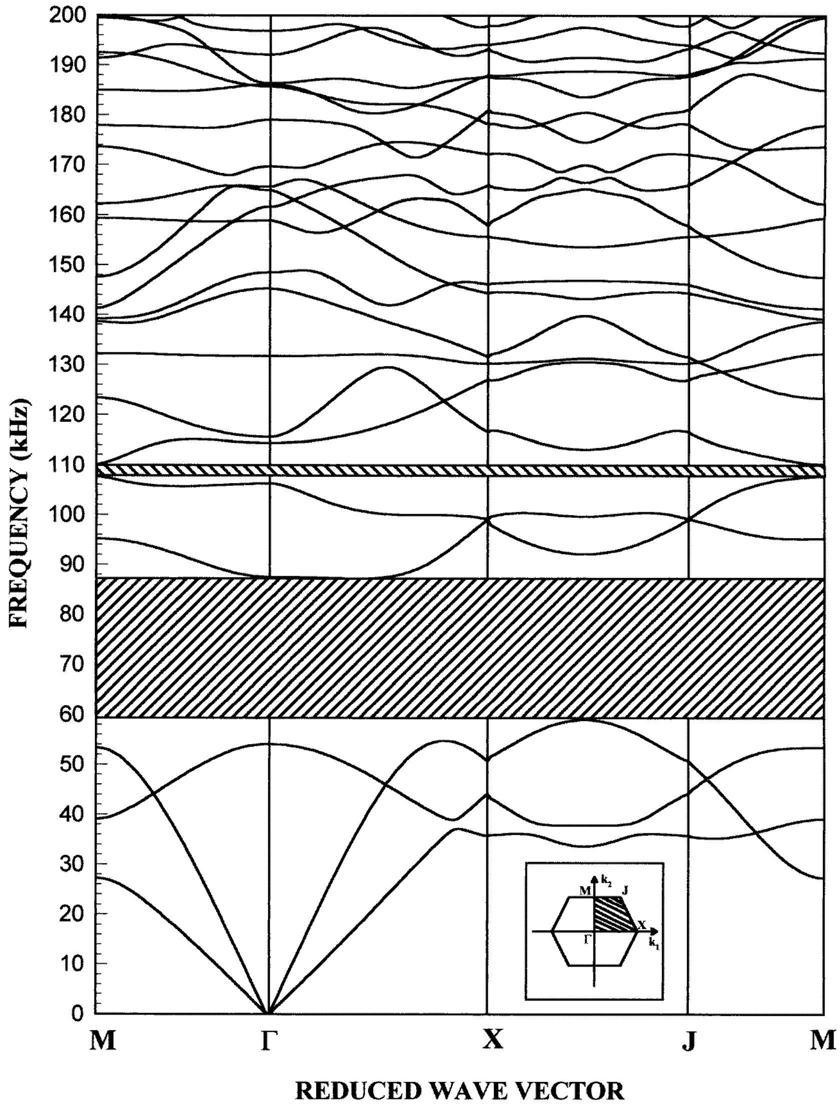

Figure 4. The same as in figure 3 for the periodic centred rectangular array of Duralumin cylinders in epoxy. The points Γ, X, J and M in the 2D Brillouin zone have reduced components (0, 0), (5/8, 0), (3/8, 1/2) and (0, 1/2).

dimensional Brillouin zone on a large number of $\boldsymbol{k}$ points. This computation confirms that the existing gaps extend throughout the Brillouin zone and are not only on its periphery. One therefore concludes that these gaps are absolute for the XY modes of vibration. The theoretical model predicts that acoustic waves of this polarization cannot propagate in this domain of frequency.

4. Discussion and conclusions

To summarize the experimental findings, the measurement of transmission spectra resulted in the observation of two gaps extending from 55 to 85 kHz and 115–125 kHz in the case of the finite size sample with a square array of cylinders. The sample with a centred rectangular array of cylinders seems to exhibit one gap extending from approximately 55 to 85 kHz. In order to compare, with more precision, the experimental measurements with the theoretical predictions, we have calculated the XY density of states, in the range $0 < \nu < 500$ kHz, for wave vectors in the direction ΓX of the first two-dimensional Brillouin zone of the square array (see figure 5(a)). This direction corresponds to the direction of the ultrasonic waves beam used in the experimental measurements on the square array. One observes, in figure 5(a), that an infinite periodic square lattice exhibits in this direction of propagation, one large region of null density of states between 58 and 90 kHz. One notes that the first low transmission region in the experimental spectrum (see figure 2(a)) overlaps with this theoretical gap. Around 120 kHz, the XY density of states is not strictly equal to zero but reaches a low value. This domain of frequency may be compared to the second low transmission region of figure 2(a) and corresponds to a local gap in the band structure. The geometry of the sample has prevented the experimental measurement of transmission along the ΓM direction. On that basis, our experimental transmission spectra do not constitute a proof that the observed forbidden bands are absolute in nature but serve only as strong evidence for the existence of such an absolute gap. One may expect that measurement along the ΓM direction should show a region of low transmission in the range 55–85 kHz to confirm that this domain of frequency is an absolute gap.

Figures 5(b) and 5(c) show the XY density of states in the directions ΓM and ΓX of the Brillouin zone of the rectangular centred array, respectively. These figures may be compared to the transmission spectra of figures 2(b) and 2(c). Figure 5(b) exhibits one large region of null density of states between 55 and 85 kHz which corresponds to the local gap in the direction ΓM of figure 4 and coincides with the region of low transmission of figure 2(b). In the direction ΓX, the same local gap is observed in figure 5(c) but this gap appears enlarged on the experimental spectrum of figure 2(c). One should stress that, in this direction, the transmitted power is strongly attenuated by the sample even for very low frequencies and it is quite difficult to define precisely the edges of the region with noise level transmission. Such a problem does not arise in the case of the square array of cylinders or in the direction ΓM of the centred rectangular array. This could be imputed to the anisotropy of the sample with the centred array of cylinders and to its finite size. Compared to face (1) of the sample the location of scatterers in this direction is very different. Moreover, one observes in figure 4, that the fourth band which gives the upper limit of the gap, is flat in the direction ΓX except in the vicinity of point X. One may think that such a flat band corresponds to vibration modes which are mainly localized inside or in the vicinity of the inclusions and do not expand too much inside the matrix. It is possible that this kind of vibration mode cannot be detected by the experimental apparatus located at the surface of the sample. This observation could explain the widening of the gap.

On the other hand, for both samples, the predicted absolute gap around 110 kHz is not clearly observed in the experimental spectra. In this range of frequency, figures 2(a) and 2(b) exhibit domains of reduced transmission but with intensity larger than the noise level. In figure 2(c), transmission increases in that latter region. We believe that the narrowness of the predicted gap may set it outside the capabilities of our equipment. Furthermore, such a narrow band resulting from the periodicity of the infinite lattice may not be seen in the case of the finite samples used in the experiment. The differences between calculated and

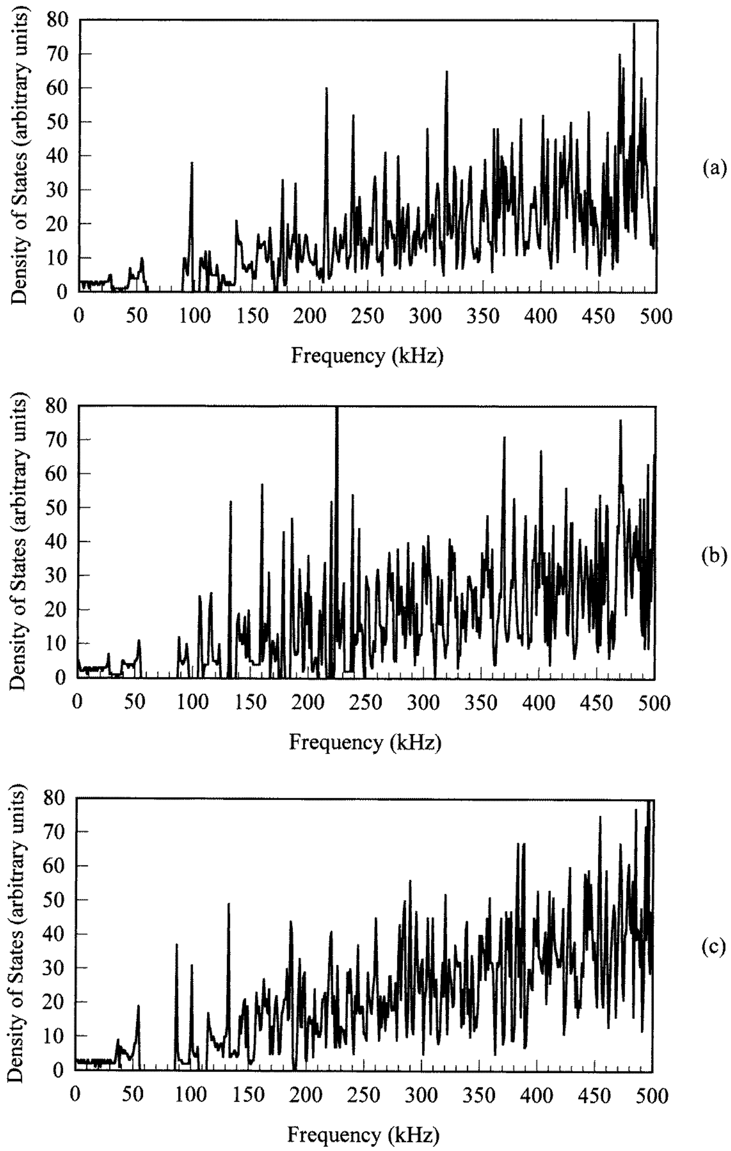

Figure 5. Calculated densities of XY states in (a) the direction ΓX of the square array, (b) the direction ΓM of the centred rectangular array and (c) the direction ΓX of the centred rectangular array.

measured spectra may also result from the fact that the XY vibrational modes have been decoupled from the Z modes in the theoretical calculation which is definitely not the case in the experimental condition. It appears necessary in order to solve these discrepancies, especially for arrays of reduced symmetry such as the centred rectangular lattice, to calculate acoustic transmission spectra for two-dimensional composite materials of finite size. Finally, some of the differences between the experimental and theoretical spectra may be, also, a consequence of the dimension of the transducers being on the order of the period of the samples. The transducer diameter is approximately 50% larger than the period of the square lattice and the centred rectangular array in the direction x_1. In the direction x_2, the period of the rectangular array exceeds the radius of the transducer by nearly 25% (see figure (1)). In particular, we anticipate the averaging procedure used to obtained compounded spectra to be valid for the square and rectangular arrays in the direction x_1 as significant beam overlap is achieved for the two probed positions. Larger uncertainties are therefore anticipated for the spectra of the centred rectangular array along the x_2 direction. This problem may be solved by using samples with smaller periods. However, this has not been possible because of the added difficulties in manufacturing such smaller composites.

In conclusion, both experimental measurements and theoretical calculations support quite clearly and unambiguously the existence of large acoustic band gaps at low frequency in 2D periodic binary solid/solid composite media. The sample with a square array shows the existence of one large gap in the transmission spectrum. This gap is in agreement with the prediction of one large absolute band gap in the theoretical band structure. The width and the frequency domain of the theoretical gap are comparable to those of the experimental one. Along one direction of propagation (ΓM), experimental measurements and theoretical predictions agree quite well for the rectangular centred array. In light of the agreements between the observed and calculated gaps, we believe that the theoretical model presents some predictive capabilities. The differences between the experimental and numerical results are compelling arguments for the development of numerical methods for the prediction of transmission spectra of finite composite systems. This will be the subject of future work.

Acknowledgments

We are grateful to D Prevost and L Manet, UFR de Physique, Université de Lille I, France for their help during the sample preparation. Thanks are also due to P François, LSPES, UFR de Physique, Université de Lille I, for the experimental measure of the speeds of sound in Duralumin and epoxy. This work is partially supported by Le Conseil Régional Nord-Pas de Calais.

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