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Propagation of acoustic waves in periodic and random two-dimensional composite media

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Transmission of acoustic waves in two-dimensional composite media composed of arrays of Duralumin cylindrical inclusions embedded in a poly vinyl chloride (PVC) matrix is studied. Experimental and theoretical results for the transmission spectrum of a periodic array of cylinders organized on a square lattice are reported. Local gaps in the first two-dimensional Brillouin zone are predicted and observed. The experimental measurements of power spectra for a random array of inclusions show a weak correlation between disorder and acoustic absorptions up to a high degree of disorder. Only in the case of highly random arrangements does the transmission spectrum diverge from the one obtained with a periodic array of inclusions.

I. INTRODUCTION

The problem of propagation of classical waves in composite media has recently received a great deal of attention. Of particular interest is the existence of gaps in the optical or acoustic band structure of periodic and random composite media. The band structure of dielectric composites is the object of extensive theoretical and experimental studies.[1] The subject of propagation of acoustic waves in inhomogeneous media has also been investigated theoretically.[2–5] These investigations focus on two-dimensional composite media composed of periodic arrays of fibers in a matrix. Under the condition of large difference in elastic properties of the matrix and fibers, these materials exhibit gaps in the acoustic band structure.

In this paper, we report a combined experimental and theoretical study of acoustic waves propagation in two-dimensional periodic and random bimaterial composites. In Sec. II we present the experimental method and the tested composite samples. The theoretical calculation of the acoustic band structure of the periodic system is reported in Sec. III. The experimental and theoretical results for the periodic and the random specimens are discussed in Sec. IV. Finally, some conclusions regarding the propagation of acoustic waves in inhomogeneous media are drawn in Sec. V.

II. EXPERIMENTAL METHOD

A. Composite systems

The periodic composite medium is constituted of an array of 25 parallel cylinders of Duralumin (Al 95%–Cu 4%–Mg 1% alloy) arranged on a 5×5 square lattice in a poly vinyl chloride (PVC) cubic matrix. The metallic cylinders have a diameter, d, of 6 mm and the lattice constant, a, is 15 mm. The dimensions of the matrix are 9 cm $\times$ 9 cm $\times$ 9 cm. The two-dimensional cross section of this specimen is illustrated in Fig. 1(a). The filling fraction of metal to polymer defined as the volume of the rods to the total volume of the structure is 12.6%. The choice of the materials is based on the strong contrast in elastic properties between Duralumin and PVC which is a necessary condition for the existence of acoustic gaps.[2] Duralumin and PVC are supposed isotropic. The mass density, ρ, the longitudinal, c_l, and transversal, c_t, speeds of sound and the elastic constants C_{11} and C_{44} of these materials are listed in Table I.

In Fig. 1(b) we have illustrated the cross section of the random composite medium. This system differs from the periodic one in that the arrangement of the parallel cylinders is now disordered.

B. Experimental setup

The ultrasonic emission source used in the experiment is a Panametrics delta broadband 500 kHz P-transducer with pulser/receiver model 500PR. The measurement of the signals was performed with a Tektronix TDS 540 oscilloscope equipped with a TD100 Data Manager. The transducers are cylindrical with a diameter of 3.175 cm (1.25 inch). The transducers are centered on the faces of the composite specimen and the nearly parallel signal is perpendicular to the Duralumin cylinders. The emission source produces compression waves (P-waves) and the receiving transducer detects only the longitudinal component of the transmitted wave.

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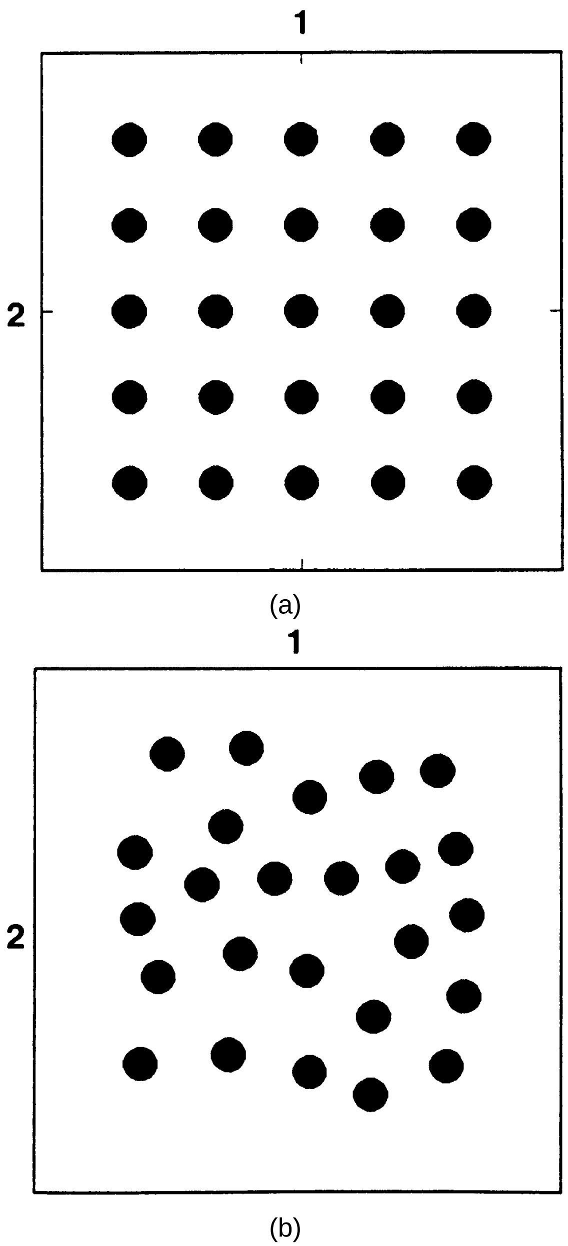

FIG. 1. Cross section of the periodic (a) and random (b) composite media. The probed faces of the specimen are labeled (1) and (2).

The measured transmitted signal is Fourier transformed to produce a power spectrum. Measurements have been performed on the faces (1) and (2) of the periodic and random composite media, as illustrated in Fig. 1.

The power spectrum for the periodic system probed on face (1) is presented in Fig. 2(a). Identical measurements have been made on face (2) of this sample. The spectra for faces (1) and (2) of the random composite medium are reported in Figs. 2(b) and 2(c).

III. THEORETICAL METHOD

We used in this paper a theoretical method developed by Kushwaha *et al.*[3–5] The two-dimensional periodic system is modeled as an array of infinite cylinders of circular cross section made of an isotropic material A (A being Duralumin) embedded in an infinite isotropic elastic matrix B (B being P.V.C.). The lattice constant is a and the filling fractions are f and $(1 - f)$ for the materials A and B, respectively ($f = 0.126$). The elastic parameters are periodic functions of the position. The mass density ρ and the elastic constants C_{ij} are ρ^A and C_{ij}^A inside the cylinders and ρ^B and C_{ij}^B in the background. It means that ρ and C_{ij} are functions of the coordinates x and y where the z axis defines the direction of the cylinders. Considering the double periodicity in the xy plane, we can write ρ and C_{ij} as Fourier series:

$$\rho(\mathbf{r}) = \rho(x,y) = \sum_{\mathbf{G}} \rho(\mathbf{G}) e^{i\mathbf{Gr}}, \tag{1a}$$

$$C_{ij}(\mathbf{r}) = C_{ij}(x,y) = \sum_{\mathbf{G}} C_{ij}(\mathbf{G}) e^{i\mathbf{Gr}}, \tag{1b}$$

where $\mathbf{r}$ is the position vector of components x and y and $\mathbf{G}$ are the reciprocal lattice vectors in the xy plane. The Fourier coefficients in Eq. (1a) take the form:

$$\rho(\mathbf{G}) = \frac{1}{A_c} \int\!\!\int d^2\mathbf{r}\, \rho(\mathbf{r}) \mathrm{e}^{-i\mathbf{Gr}}, \tag{2}$$

where the integration is performed over the unit cell of area $A_c = a^2$.

For $\mathbf{G} = 0$, Eq. (2) gives the average density:

$$\rho(\mathbf{G} = 0) = \overline{\rho} = \rho^A f + \rho^B (1 - f). \tag{3a}$$

For $\mathbf{G} \neq 0$, Eq. (2) may be written as:

$$\rho(\mathbf{G} \neq 0) = (\rho^A - \rho^B) F(\mathbf{G}) = (\Delta\rho) F(\mathbf{G}), \tag{3b}$$

where $F(\mathbf{G})$ is the structure factor given by:

$$F(\mathbf{G}) = \frac{1}{A_c} \int\!\!\int_A e^{-i\mathbf{Gr}}\, d^2\mathbf{r}. \tag{3c}$$

In Eq. (3c), the integration is performed only on material A. For a square array of infinite cylinders of radius r_o, the structure factor is $F(\mathbf{G}) = 2f[J_1(Gr_o)/Gr_o]$ where J_1 is the Bessel function of the first kind. In an entirely similar way, Eq. (1b) gives:

$$\begin{cases} C_{ii}(\mathbf{G} = 0) = \overline{C_{ii}} = C_{ii}^A f + C_{ii}^B (1 - f) & \text{(4a)} \\ C_{ii}(\mathbf{G} \neq 0) = (C_{ii}^A - C_{ii}^B) F(\mathbf{G}) = (\Delta C_{ii}) F(\mathbf{G}), & \text{(4b)} \end{cases}$$

with $i = 1$ or 4.

Let us now give the equations of motion in the composite material, remembering that the elastic constants and the mass density are position dependent[6]:

$$\rho(\mathbf{r}) \frac{\partial^2 u_i}{\partial t^2} = \nabla \cdot [C_{44}(\mathbf{r}) \nabla u_i] + \nabla \cdot \left[C_{44}(\mathbf{r}) \frac{\partial \mathbf{u}}{\partial x_i} \right] + \frac{\partial}{\partial x_i} [(C_{11}(\mathbf{r}) - 2C_{44}(\mathbf{r})) \nabla \cdot \mathbf{u}], \tag{5}$$

TABLE I. Acoustic and elastic properties of PVC (Ref. 8) and Duralumin (Ref. 9) used in this study.

	ρ ($kg \cdot m^{-3}$)	C_l ($m \cdot s^{-1}$)	C_t ($m \cdot s^{-1}$)	$C_{11} = \rho \cdot C_l^2$ (10^9 $N \cdot m^{-2}$)	$C_{44} = \rho \cdot C_t^2$ (10^9 $N \cdot m^{-2}$)
PVC	1389	2530	1220	8.891	2.067
Duralumin	2790	6320	3130	111.439	27.333

where **u** stands for the position and time dependent displacement vector $\mathbf{u}(\mathbf{r}, t)$.

For wave propagation in the xy plane, one can introduce a wave vector $\mathbf{K}(K_x, K_y)$ (which means $K_z = 0$) and use the Bloch theorem to write the displacement field:

$$\mathbf{u}(\mathbf{r}, t) = e^{i(\mathbf{Kr} - \omega t)} \sum_{\mathbf{G}} \mathbf{u}_{\mathbf{K}}(\mathbf{G}) e^{i\mathbf{Gr}} , \tag{6}$$

where ω is the wave circular frequency. In this case the vibrations polarized parallel to the z axis become decoupled from those in the xy plane. The equations of motion for the former modes are written as:

$$\begin{aligned} &[\overline{C}_{44}(\mathbf{K} + \mathbf{G})^2 - \overline{\rho}\omega^2] u_{\mathbf{K}}^z(\mathbf{G}) \\ &\quad + \sum_{\mathbf{G}' \neq \mathbf{G}} [(\Delta C_{44})(\mathbf{K} + \mathbf{G}) \cdot (\mathbf{K} + \mathbf{G}') \\ &\qquad - (\Delta\rho)\omega^2] F(\mathbf{G} - \mathbf{G}') u_{\mathbf{K}}^z(\mathbf{G}') = 0 , \end{aligned} \tag{7}$$

whereas the latter modes are governed by the equation:

$$\begin{aligned} &[\overline{C}_{44}(\mathbf{K} + \mathbf{G})^2 - \overline{\rho}\omega^2] \mathbf{u}_{\mathbf{K}}^{\mathbf{T}}(\mathbf{G}) \\ &\quad + (\overline{C}_{11} - \overline{C}_{44})(\mathbf{K} + \mathbf{G})(\mathbf{K} + \mathbf{G}') \cdot \mathbf{u}_{\mathbf{K}}^{\mathbf{T}}(\mathbf{G}) \\ &\quad + \sum_{\mathbf{G}' \neq \mathbf{G}} F(\mathbf{G} - \mathbf{G}')\{(\Delta C_{44})[(\mathbf{K} + \mathbf{G}) \\ &\quad \cdot (\mathbf{K} + \mathbf{G}')\mathbf{u}_{\mathbf{K}}^{\mathbf{T}}(\mathbf{G}') + (\mathbf{K} + \mathbf{G}')(\mathbf{K} + \mathbf{G}) \\ &\quad \cdot \mathbf{u}_{\mathbf{K}}^{\mathbf{T}}(\mathbf{G}') - 2(\mathbf{K} + \mathbf{G})(\mathbf{K} + \mathbf{G}') \cdot \mathbf{u}_{\mathbf{K}}^{\mathbf{T}}(\mathbf{G}')] \\ &\quad + (\Delta C_{11})(\mathbf{K} + \mathbf{G}) \cdot (\mathbf{K} + \mathbf{G}') \cdot \mathbf{u}_{\mathbf{K}}^{\mathbf{T}}(\mathbf{G}') \\ &\quad - (\Delta\rho)\omega^2 \mathbf{u}_{\mathbf{K}}^{\mathbf{T}}(\mathbf{G}')\} = 0 , \end{aligned} \tag{8}$$

where $\mathbf{u}^{\mathbf{T}} = u_x\mathbf{x} + u_y\mathbf{y}$.

Equations (7) and (8) are two infinite sets of linear equations where the unknowns are the Fourier components of the displacement field. In the practice of the numerical calculation, only a finite number of **G** vectors are, of course, taken into account. The determinants of these systems of equations must vanish, which conditions yield the band structure $\omega_n(\mathbf{K})$. The eigenmodes of Eq. (7) correspond to transverse vibrations ($\mathbf{u} = u^z\mathbf{z} \perp \mathbf{K}$) and will be called Z-modes. On the other hand, the eigenvalues of Eq. (8) describe modes of vibration for which **u** and **K** are coplanar vectors in the xy plane and will be denoted XY-modes. These modes are also named coupled longitudinal-transverse vibrations in the literature.[4]

In this paper, we solve numerically for the eigenvalues of the two-dimensional XY-modes along high symmetry directions in the first two-dimensional Brillouin zone taking into account 338 **G** vectors. The band structure of the XY-modes in the periodic Duralumin/PVC (the physical characteristics of these materials are listed in Table I) composite medium is reported in Fig. 3. This band structure does not show any gap extending throughout the Brillouin zone but only local gaps in some specific directions.

IV. DISCUSSION OF RESULTS

A. Periodic composite system

The transmitted power spectrum in Fig. 2(a) shows three significant absorptions at approximately 295, 382, and 488 kHz. States above 700 kHz cannot be resolved experimentally due to limits in detectability. Although the emission source produces longitudinal waves only, scattering in the specimens produces mixed XY-modes. In first approximation, in order to compare the experimental measurements with the computed band structure, we have calculated the XY density of states for wave vectors in the direction ΓX of the first two-dimensional Brillouin zone. The density of states calculation is limited to the first nine bands in the band structure. In Fig. 4 we have superimposed the power spectrum and the density of states. The band structure in the ΓX direction possesses a local gap showing in the density of states at a frequency of 507 kHz. This frequency appears to be in reasonable agreement with the observed absorption at 488 kHz. The fact that this absorption does not go to zero power may be explained on the basis of only approximate two-dimensional propagation of acoustic waves in the experimental medium as well as boundary effects from the finite sample. Moreover, the receiving transducer detects only the longitudinal part of the transmitted waves. The observed absorption at the frequency of 295 kHz may be assigned to the drop in density of states between 250 and 300 kHz corresponding to the local gap between the first and second band at the point X.

Another absorption in the experimental spectrum at the frequency 382 kHz does not appear to have a counterpart in the calculated density of states. This frequency, however, seems to correspond to a crossing between the second and the third band in the band structure at a frequency of 364 kHz. We speculate that a local gap near the observed absorption may be produced by coupling between the second and the third band. Anti-

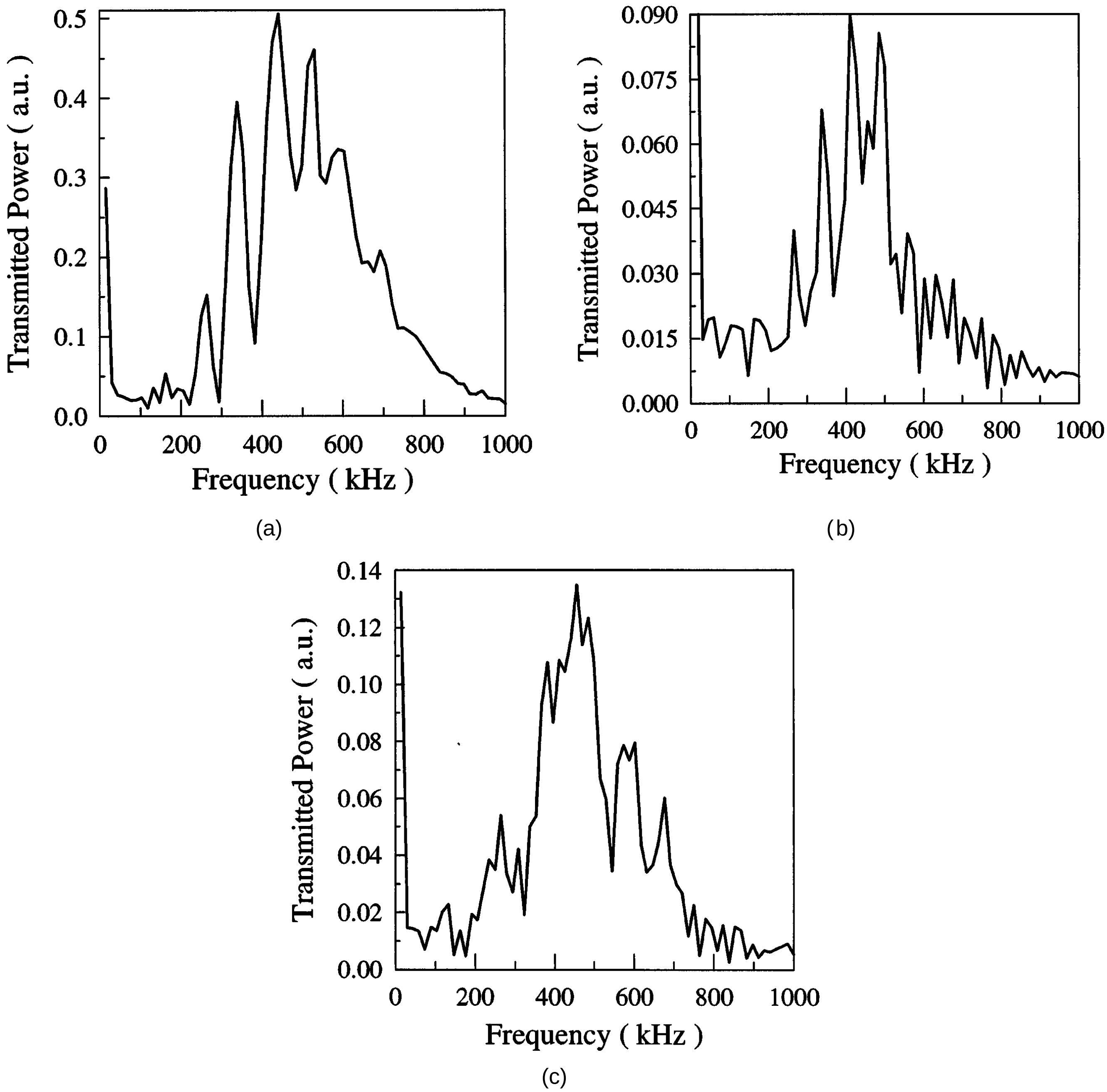

FIG. 2. Power spectrum for the periodic composite system (a) and the random composite system measured perpendicular to the face (b) and (c). The units for power are arbitrary. The maximum signal intensity of the random sample is approximately 20% (b) and 30% (c) of the maximum signal of the periodic composite.

crossing of these bands may be due to perturbations from anharmonic effects or internal stresses in the specimen.

B. Random composite system

The power spectra for the random system measured perpendicular to the faces (1) and (2) show differences. However, we note a striking similarity between the spectrum for the periodic system and that of the random system of Fig. 2(b). This latter spectrum shows again three significant absorptions near 290, 382, and 483 kHz. In contrast, the spectrum 2(c) does not resemble the periodic spectrum. This result is suggestive of different degrees of randomness in the two probed directions. To quantify the nature of randomness in both directions we introduce an order parameter as follows:

$$O = \frac{1}{n} \sum_{i}^{n} \cos(\mathbf{G}\mathbf{r}_{\mathbf{i}}) , \quad (9)$$

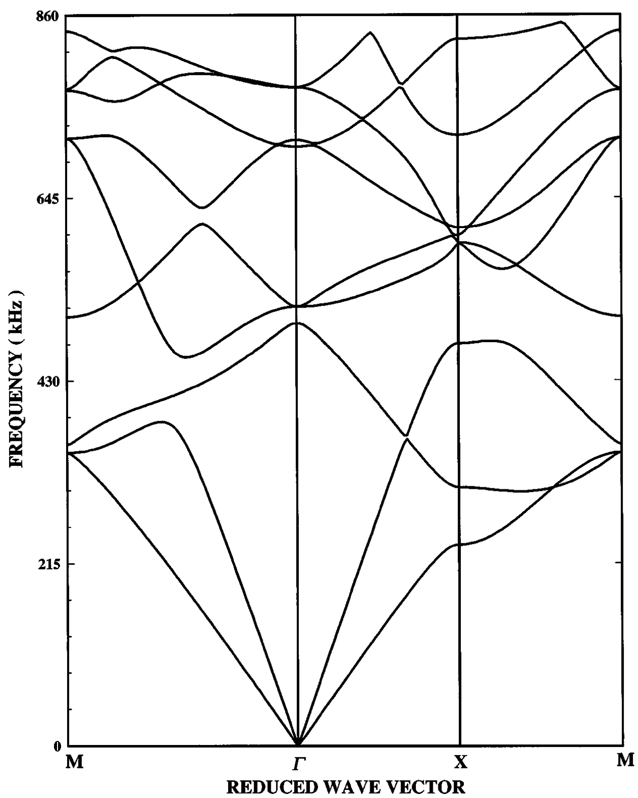

FIG. 3. Band structure for the two-dimensional XY-modes in periodic Duralumin/PVC composite. The reduced wave vector is defined as $\mathbf{K}a/2\pi$ where $\mathbf{K}$ stands for the two-dimensional wave vector. The points Γ, X, M in the 2-D Brillouin zone have components $(0, 0)$, $(1/2, 0)$, and $(1/2, 1/2)$, respectively.

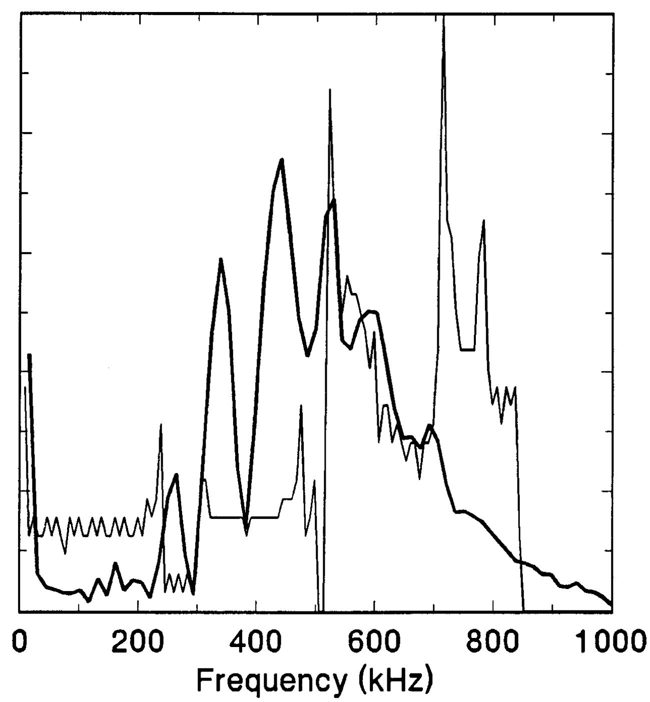

FIG. 4. Experimental power spectrum (thick line) and calculated density of states (thin line) for the periodic composite medium. The vertical axis is in arbitrary units.

where $\mathbf{r_i}$ is a two-dimensional positional vector of cylinder i. The sum is taken over those n cylinders within the cross section of the transducers. This cross section is defined as a region approximately 3 cm wide centered on the probed face and 9 cm deep. We calculate the values of the order parameter, O_1 and O_2, in the directions perpendicular to the faces (1) and (2) by choosing $\mathbf{G_1} = (0, 2\pi/a)$ and $\mathbf{G_2} = (2\pi/a, 0)$.

In the case of Fig. 2(b), or probe perpendicular to face (1), we obtain the following results: $O_1 = 0.16$ and $O_2 = 0.24$ with 12 cylinders contributing to wave scattering. The order parameters for the case of Fig. 2(c), or probe perpendicular to face (2), are $O_1 = 0.0$ and $O_2 = 0.16$ with 14 cylinders in the transducer cross section. These values support the differences between the experimental spectra for the random specimen. In both cases, the order parameter in the direction of wave propagation is the same. The major difference is found in the ordering of the cylinders in the direction perpendicular to wave propagation. The measurements suggest that nearly 80% disordering in the arrangement of the cylinders perpendicular to the direction of wave propagation still produces a transmitted acoustic signal characteristic of a periodic array of cylinders. We note from our experiment that disorder perpendicular to the direction of wave propagation in excess of 80% may produce drastic modifications in the acoustic transmitted signal. This result seems to be in accord with the observations made by Achenbach and Kitahara[7] on the existing correlation between disorder and harmonic waves attenuation in composite systems.

Moreover, the difference in spectra 2(b) and 2(c) shows that the experimental absorption at 382 kHz in the periodic sample cannot be assigned to boundary effects due to the finite size of the specimen but indeed is related to periodicity and therefore to the band structure.

V. CONCLUSIONS

We have conducted a joint experimental and theoretical study of acoustic waves propagation in periodic and random two-dimensional composite materials. This study was motivated by the recent interest devoted to finding classical wave gaps in inhomogeneous media.

Our experimental investigation of a composite medium composed of parallel Duralumin cylinders arranged on a square lattice embedded in a PVC matrix shows the existence of strong acoustic absorptions. These absorptions have been assigned to local gaps in the theoretically calculated band structure for XY-modes. This band structure was calculated within the approximation of two-dimensional wave propagation. Despite the presence of local gaps the periodic composite medium Duralumin/PVC does not exhibit acoustic band gaps extending throughout the Brillouin zone.

The experimental study of a composite medium containing metallic cylinders arranged in a random fashion has shown the existence of a correlation between disorder and acoustic wave transmission. The general features of the transmitted power spectrum appear to be insensitive to disorder up to a very large degree of disorder. In the case studied, a composite medium with a degree of disorder of nearly 80% as measured by a two-dimensional order parameter shows striking similarities with the periodic composite system in its transmitted spectrum. On the other hand, the power spectrum loses the characteristics of the periodic system when the order parameter drops well below 20%.

We are currently examining other candidate composite media that are predicted theoretically to possess gaps throughout the Brillouin zone. These media differ from the one studied here by their larger filling fraction and greater contrast in elastic properties of the inclusions and matrix. Of particular interest is the influence of the geometry of the inclusions as well as the nature of their arrangement within the matrix.

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