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Transmission of acoustic waves through waveguide structures in two-dimensional phononic crystals

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Transmission of acoustic waves through waveguide structures in a two-dimensional phononic crystal made of PVC cylinders arranged on a square array in air is studied. We investigate particularly the transmission through a perfect rectilinear waveguide and waveguides coupled with a side branch resonator. A rectilinear wave guide obtained by removing one row of cylinders in the perfect phononic crystal can support one or several modes falling in the complete band gap of the phononic crystal. The effect of a side branch resonator is to induce zeros of transmission in the spectrum of the perfect guide that appear as narrow dips with frequencies depending upon the shape of the resonator. Most of these theoretical predictions obtained by using the plane wave expansion (PWE) and the finite difference time domain (FDTD) methods are compared to experimental measurements made in the audible frequency range.

1 Introduction

Phononic crystals (PC's) are inhomogeneous materials made of two- or three-dimensional periodic arrangements of inclusions embedded in a matrix [1, 2]. PC's possess complete band gaps in their acoustic transmission spectrum (i.e. gaps independent of the direction of propagation of an incident wave). The removal of inclusions along some pathway in the PC produces acoustic waveguides [3, 4]. Acoustic waves that would not propagate otherwise in a PC can be guided with minimal loss along such waveguides. Furthermore, the passing band of a guide can be altered by attaching resonators to its side. For instance, recent theoretical studies have shown that side branch resonators obtained by removing additional inclusions in a direction perpendicular to a linear waveguide in a two-dimensional PC, induce zeros of transmission in the spectrum of the unperturbed guide [4]. In support of these studies, we prove experimentally, in this paper, the existence of such zeros of transmission associated with the presence of resonators in the vicinity of a waveguide. The experimental results are supplemented by numerical calculations of dispersion curves and transmission coefficients based on the plane wave expansion (PWE) [1] and the finite difference time domain (FDTD) methods [5, 6]. We present the structures and the experimental setup in section 2. Section 3 contains the results which consist essentially of measured and calculated transmission spectra for a linear waveguide in a 2D PC and a waveguide with a side branch resonator of variable length. Conclusions are drawn in section 4.

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2 System and experimental setup

The basic experimental system is a two-dimensional solid/fluid PC composed of 18x18 Polyvinyl Chloride (PVC) cylinders in air. The cylinders, of radius r = 1.29 cm, are one meter long, parallel to the Z direction of the (O, X, Y, Z) Cartesian coordinates system (see inset in Fig. 1) and are arranged on a square array with lattice parameter a = 2.7 cm. We have chosen a PVC/air PC with lattice parameter in the centimeter range to achieve acoustic band gaps in the audible range of frequency [1]. Acoustic waveguides and resonators are easily created in this structure by removing cylinders. More specifically, we have constructed, a linear waveguide (Fig. 1(a)) and two resonators of different lengths coupled to a linear guide (Figs. 1(b) and 1(c)). We measured the transmission across the PC along the guides with a speaker connected to a function generator and a microphone whose frequency response lies in the range [40 Hz–12 kHz]. The transmitted signal recorded by the microphone is detected with a tracking generator coupled to a spectrum analyzer. The speaker and microphone are placed against the walls of the structures in the same plane perpendicular to the cylinders. For each system, two measurements are conducted with and without the structure. The difference between these frequential signals is calculated to substract any background effect. We have verified theoretically and experimentally that the PC presents stop bands extending from 2.8 to 10 kHz along the ΓX direction of the square irreducible Brillouin zone (i.e. waves propagating along the Y direction) and from 4 to 10 kHz along the ΓM direction (waves propagating along the diagonal of the square array). These stop bands overlap between 4 and 10 kHz. This frequency domain corresponds to the complete forbidden band of the structure.

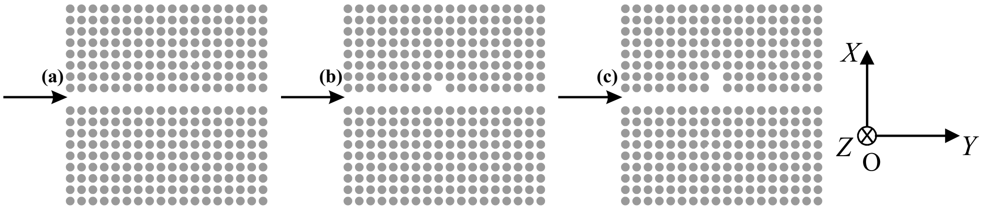

Fig. 1 Cross sections of the phononic crystals with (a) the perfect linear waveguide, (b) a stub of length equal to one period attached vertically to the waveguide, (c) a stub two periods long. The arrow indicates the direction of propagation of the incident longitudinal wave.

3 Results

3.1 Linear waveguide

Figure 2(a) presents the band structure of the 2D PC with a linear waveguide. These dispersion curves were obtained numerically using the PWE method with a supercell of 7 periods and by imposing the condition of elastic rigidity to the solid inclusions. The supercell contains 7 unit cells, one of which is filled with air only. The supercell is also repeated periodically in the X direction leading to a stack of waveguides separated by 7 periods in this direction. This separation is sufficient to avoid coupling between neighbouring guides. We assumed the solid as perfectly rigid in order to resolve the difficulties encountered when the PWE method is used for calculating the band structure of PC’s made of solid scatterers surrounded by a fluid matrix. On the practical side, we replace the solid by a fluid with equivalent longitudinal speed of sound and density [7, 8]. In comparison to air, this solid is nearly rigid. The dispersion curves numbered 6, 7, 8 and 9 are related to localized modes in the straight waveguide and fall in the stop band of the perfect PC. The measured and calculated (using the FDTD method) transmission spectra of the linear waveguide are presented in Figs. 2(b) and 2(c), respectively. Experimentally, the waveguide permits transmission of waves that otherwise would be forbidden in the perfect PC. There are two waveguide passing bands with frequency intervals [2.4, 5.6] kHz and [6.8, 8.5] kHz. Transmission of waves with frequency in these intervals takes place along the waveguide without any loss. Stop bands still exist for frequencies between 5.6 and 6.8 kHz and 8.5 and 10 kHz. The experimental spectrum is confirmed by the FDTD calculation. Indeed Fig. 2 (c) shows two passing bands from 0 to 5.5 kHz and

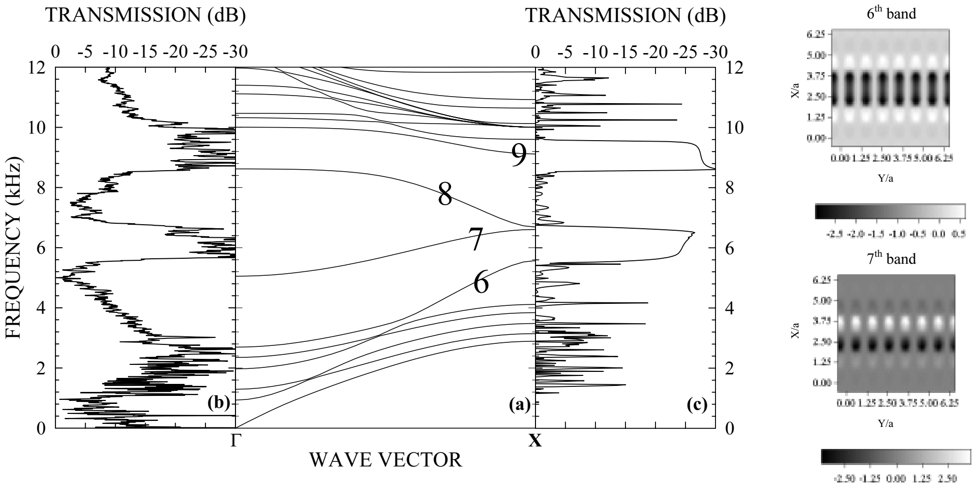

Fig. 2 (a) PWE band structures of a PVC/air phononic crystal containing a straight waveguide along the Y direction (see Fig. 1(a)). Experimental (b) and theoretical (c) transmission spectra along the linear waveguide. The bands numbered 6, 7, 8 and 9 in Fig. 2(a) are associated with waveguides modes. Bands 7 and 9 are "*deaf bands*" and do not contribute to the transmission. The maps represent the pressure fields inside the PVC/air phononic crystal. The grey scale indicates the relative amplitude of the pressure field. Modes are shown at point X for bands 6 and 7.

6.8 to 8.5 kHz, separated by a region with low transmission that extends from 5.5 to 6.8 kHz. The gap of the perfect PC persists for frequencies in the range [8.5, 9.6] kHz. A comparison between the transmission spectra and the band structure indicates that the dispersion curves labelled 7 and 9 in Fig. 2(a) do not contribute to the transmission. This singular effect may be explained by the symmetry of these modes which can be observed from the pressure field inside the structure [9]. The right panel of Fig. (2) illustrates the pressure field pattern corresponding to the 6th and 7th bands at the X point of the irreducible Brillouin zone. It is important to note that the 6th mode has a symmetry readily excitable by an incident wave of longitudinal polarization such as a sonic wave. In contrast, the 7th mode is anti-symmetric with respect to the symmetric plane of the waveguide. Consequently, this antisymmetric mode cannot be excited by a longitudinal incident wave and will not contribute to the transmission. Similar calculations proved that the 8th (resp. 9th) band corresponds to a symmetric (resp. antisymmetric) mode. Such antisymmetric modes, named "*deaf bands*", were reported previously for perfect PC's [9]. Let us stress that the deaf modes reported in this paper are not modes of the perfect crystal but waveguide modes. One may also observe that the acoustic pressure field extends significantly beyond the bounds of the waveguide, showing that the waveguide modes are not strictly confined inside the waveguide.

3.2 Linear waveguide with side branch resonator

The effect of a side branch resonator on the transmission spectrum of the waveguide is illustrated in Fig. 3. The removal of a single cylinder adjacent to the waveguide produces a resonator of nominal length *a*. The calculated transmission spectrum in Fig. 3(a) retains the general characteristics of the linear waveguide (dotted line) with two additional features. Narrow dips appear in the calculated transmission spectrum at two frequencies, namely 4.7 and 7.5 kHz. These reductions in transmission are similar in nature to those observed in a recent theoretical study of waveguides with side branch resonators in water/air PC's [4]. The transmission in the waveguide is significantly altered due to interference phenomena between the acoustic modes of the guide and those of the resonator. The characteristics of the experimental spectrum of the guide with resonator of length *a* are best seen by calculating the difference between the transmission along the guide with resonator and that of the perfect linear guide as reported in the insets of Fig. 3(b). A small depression in transmission occurs in the first passing band of the linear guide at 4.85 kHz. This agrees quite well with the theoretically predicted dip at 4.7 kHz (see Fig. 3(a)). In the range [7, 8.5] kHz, the experimental transmission spectrum exhibits two depressions around 7.5

kHz and 8.2 kHz. The first depression is in accordance with the one observed in the theoretical transmission, but the feature at 8.2 kHz has no analog in Fig. 3(a). Nevertheless, one notes that this feature appears in the very near vicinity of the edge of the second waveguide passing band and this renders its analysis very difficult. Lengthening the resonator increases the number of resonant modes and therefore the number of narrow dips in the transmission spectrum of the guide with resonator. For instance, in the case of Fig. 3(c) where the resonator is twice as long as in Fig. 3(a), the theoretical transmission spectrum exhibits four narrow dips in transmission at 3.95, 7.3, 9.7 and 9.8 kHz. The experimental spectrum of the guide with a resonator of length 2*a* possesses a significant reduction in transmission at 4.1kHz in very good agreement with the first calculated dip (see Fig. 3(c)). Two additional and well-defined depressions in transmission are observed in the second passing band of the guide at the frequencies 7.4 and 8.2 kHz. As in the previous case, the first of these two features is in good agreement with the theoretical predictions but the second one, which appears in the near vicinity of the stop band has no equivalent in Fig. 3(c). Because the second stop band in the experimental spectrum (see Fig. 3(d)) is wider than the theoretically predicted transmission band gap, the dips calculated around 9 kHz cannot be observed experimentally. A comparison between Figs. 3(b) and 3(d) reveals that the influence of the resonator on the waveguide transmission is much more pronounced with a longer resonator. For instance, the first depression in transmission which occurs around 4.5 kHz in these two figures is much more important with a resonator of nominal length 2*a* than *a*. As noted before, the waveguide modes extend significantly beyond the physical bounds of the guide. The interference between these modes and those of a short resonator are therefore anticipated to be weak.

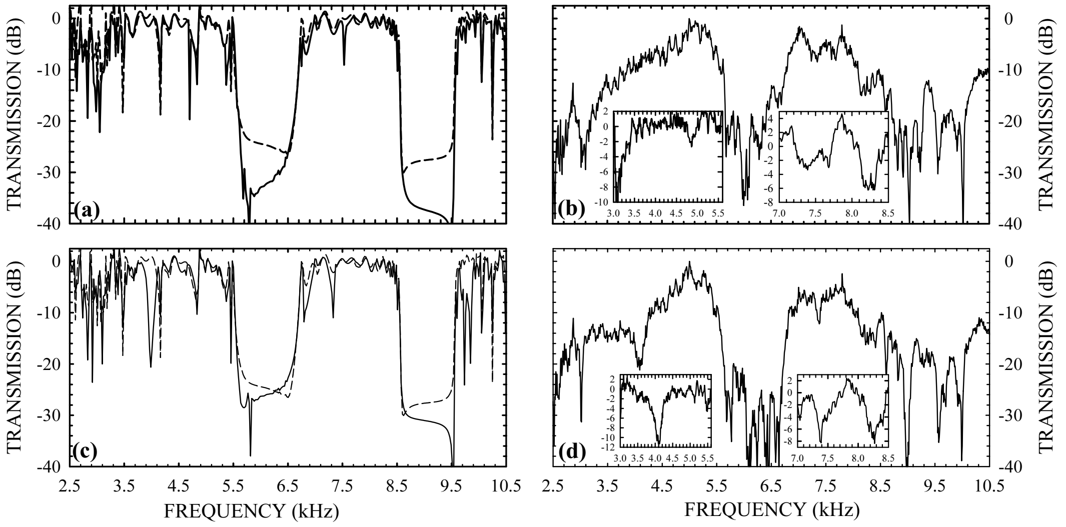

Fig. 3 Theoretical ((a),(c)) and experimental ((b),(d)) transmission spectra along the linear waveguide with a side branch of nominal length *a* ((a) and (b)) and along the linear waveguide with a side branch of nominal length 2*a* ((c) and (d)). The insets in (b) and (d) represent the difference between the transmission in the guide with resonator and the transmission in the linear waveguide in the range of frequency associated with the passing bands of the linear guide. The dashed line in (a) and (c) represents the computed transmission along the linear waveguide of Fig. 2(c).

4 Conclusion

We have demonstrated the possibility of resonant filtering in a linear PC waveguide with one single side branch resonator. Frequency filtering takes place by reduction of the transmission at specific frequencies within the passing band of the waveguide. These frequencies depend on the length of the resonator. The experimental results obtained with a basic experimental setup are in fair agreement with theoretical calculations based on the FDTD method, especially at the lowest frequencies studied. The structures presented in this paper may serve as element in the design of devices for the treatment of acoustic signals such as filtering or demultiplexing.

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