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Transmission and dispersion modes in phononic crystals with hollow cylinders: application to waveguide structure

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In this paper, we present a theoretical analysis of the propagation of acoustic waves through phononic crystals constituted of square array of hollow cylinders of steel immersed in water. The study of the transmission along the principal high-symmetry direction, ΓX of the Brillouin zone reveals the presence of a Narrow Pass Band (NPB) falling inside a wide band gap. Nevertheless the band structure displays two NPB in the same frequency range. Symmetry arguments, based on the calculation of the displacement field, shows that only the lowest NPB contributes to the transmission. The two NPB transform into two wider bands along the ΓM direction of the Brillouin zone, but now only the highest branch contributes to the transmission when a normally incident wave is launched onto the phononic crystal. Finally, we show that, by inserting hollow cylinders as a line defect inside a phononic crystal of filled steel cylinders, one can realize selective frequency waveguides that may be useful for guiding and multiplexing applications.

During the last decade, several works have been devoted to the study of the propagation of acoustic waves in the so-called phononic crystals, made of two or three-dimensional [1, 2] periodic repetitions of different solid or fluid constituents, which exhibit large contrast between their elastic constants and/or mass densities. These elastic periodic composites can exhibit large acoustic band gaps where the propagation of phonons is forbidden. The existence of absolute band gaps was predicted theoretically [3] prior to being demonstrated experimentally in various phononic crystals [4]. Recent studies have been concerned with manipulating the propagation of sound through channel (or waveguides) created inside the phononic crystal that can be considered as phononic circuits [5]. These waveguides can interest engineering applications from transducer technology to guidance, filtering and wavelength division multiplexing of acoustics waves [6, 7].

Considering solid/fluid phononic crystals, we have described in a recent paper [5] a class of two dimensional (2D) acoustic band gap materials that incorporates tunable narrow passing bands (NPB) in their gaps. The frequency of the passing band is controlled by modifying the geometry of the cylindrical inclusions, with little change of the location and form of the transmission gap. This class of phononic crystals provides a means to design selective acoustic waveguides with potential filtering and demultiplexing capabilities. To name one, we have studied a heteroradii waveguide constituted of hollow cylinders with alternating inner radii. This device permits the transmission of two superposed NPB with dif-

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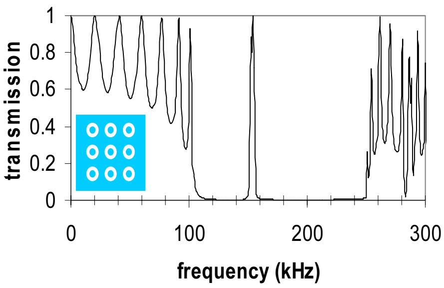
ferent frequencies. The main goal of this paper is to compare the transmission coefficient obtained on a finite size crystal made of hollow cylinders with their corresponding dispersion curves. We present a theoretical calculation of the displacement field that allows the interpretation of the differences between the transmission and the dispersion curves. Then, we discuss some applications of waveguides constituted by hollow cylinders in a phononic crystal made of filled cylinders. All calculations are made using the finite difference time domain (FDTD) method which solves the elastic wave equation by discretizing time and space and replacing derivatives by finite differences [8]. We limit the model to elastic displacements, velocities and stress fields in XY plane perpendicular to the cylindrical inclusions.

Our calculations of transmission and dispersion curves are performed for 2D phononic crystals of square geometry composed of hollow steel cylinders in water matrix (see insets in Fig. 1). The lattice parameter of the square mesh of cylinders is a = 5 mm. The inclusions are hollow cylinders with an outer radius r_e = 2.3 mm and an inner radius r_i = 1.2 mm. The longitudinal and transverse speeds of sound in steel are taken as 5825 and 3226 m/s. The longitudinal speed of sound in water is 1490 m/s. The density of steel and water are 7.78 and 1 g/cm^3, respectively.

To calculate the transmission coefficients, the model system is built of three parts. The finite phononic composite occupies the central region and is sandwiched between two homogeneous parts filled by water. The composite region is composed of five periods in the direction of propagation, Y, and one period in direction X along which periodic conditions are applied. Absorbing Mur conditions are imposed at the free ends of the homogeneous media. A broad band travelling wave packet is launched in the first homogeneous region. The signal transmitted is recorded at the end of the second homogeneous region and integrated along its width. The Fourier transform of the transmitted signal normalized to the Fourier transform of a signal propagating through a homogeneous water system of the same physical dimensions as the model composite yields a transmission coefficient.

Figure 1a shows the transmission coefficient through the perfect phononic crystal containing hollow cylinders of steel immersed in water, in the ΓX direction, as a function of frequency. The transmission spectrum exhibits a gap from 110 to 250 kHz and exhibits inside that stop band a NPB and with a central frequency that occurs at 155 kHz. The most remarkable feature is the possibility of tuning the frequency of the NPB by varying the inner radius of the tubular cylindrical inclusions [5]. We present in Fig. 1b the transmission spectrum in the ΓM direction of the Brillouin zone. In the frequency range of 100 to 240 kHz, this spectrum exhibits also one pass band but much larger than the NPB displayed in Fg. 1a. The selective NPB is then available only in the ΓX direction of the Brillouin zone.

(a)



(b)

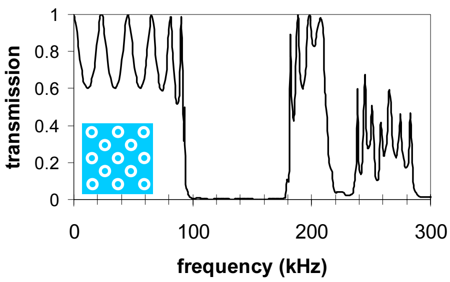


Fig. 1 Transmission spectra of the phononic crystal constituted of hollow steel cylinders in water medium, (a) for the ΓX direction and (b) for the ΓM direction of the Brillouin zone.

The above results can be quantitatively compared with the dispersion curves. To this end, the FDTD method has been adapted to the calculation of dispersion relations of acoustic waves in 2D phononic lattices [9]. Owing to the periodicity within the (X,Y) plane, the lattice displacement and the stress tensor take the forms satisfying the Bloch theorem.

In Fig. 2, we plot the band structure for the case of hollow cylinders embedded in water. In the frequency range of 100 to 240 kHz, there are two nearly flat bands along the ΓX direction which transform into two bands more dispersed along ΓM. In the transmission spectra of Fig. 1, only the lowest branch (referred to as 1st) is visible in the ΓX direction and the highest branch (referred to as 2nd) along the ΓM direction.

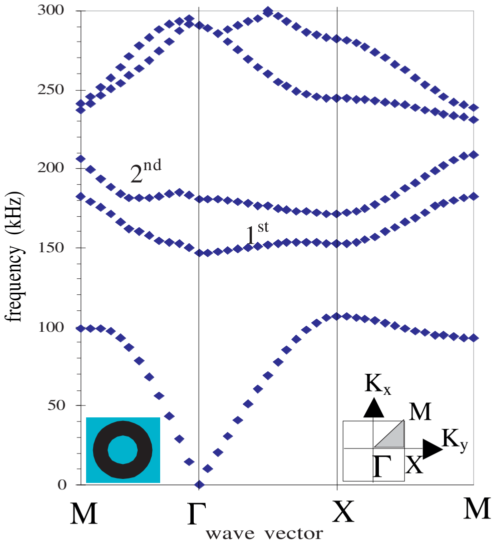


Fig. 2 Dispersion curve of the phononic crystal constituted of hollow steel cylinders in water medium. The grey area delimit the wide frequency domain as defined in the transmission curves and containing the NPB.

This apparently contradictory result can be explained by symmetry arguments. To this end, we have studied the displacement field associated with each branch in the vicinity of the Γ point of the Brillouin zone, either along ΓX or ΓM direction. The results for the lowest branch are displayed in Fig. 3.

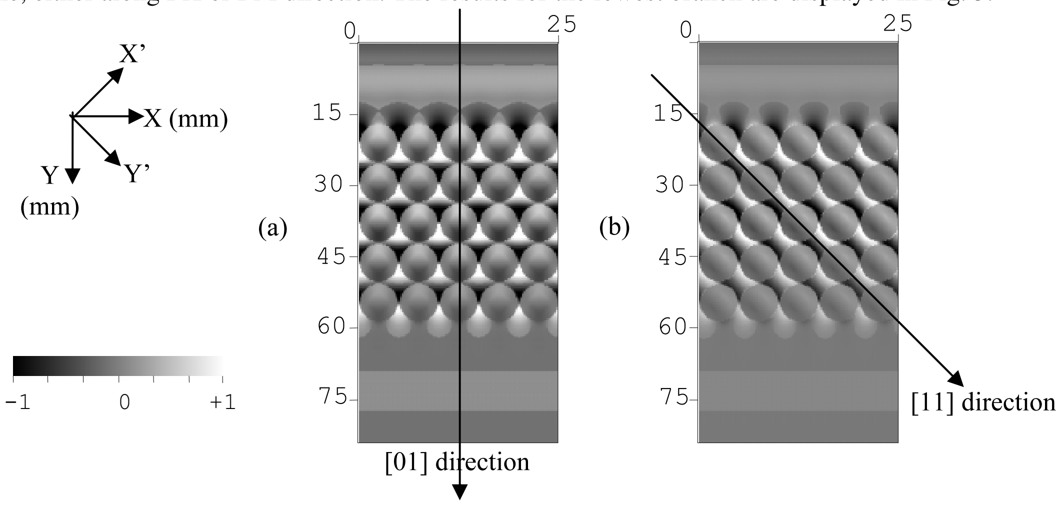


Fig. 3 Displacement fields of the first mode of the dispersion curve in the vicinity of the Γ point for a incident plane wave incoming (a) from the [01] and (b) from the [11] direction. The first one is a symmetric mode and the second an antisymmetric one.

In Fig. 3a, we have sketched the U_Y component of the displacement field for a normal incident wave propagating along the [01] (or Y) direction at the frequency of 151 kHz. One can notice the symmetry of this field with respect to the Y axis. The same symmetry is obtained for the component U_X of the displacement (not shown here). Now, from these components U_X and U_Y, we have constructed the two combinations $U_{X'} = U_X – U_Y$ and $U_{Y'} = U_X + U_Y$ that represent the components of the displacement field along the X' = [1,–1] and Y' = [1,1] directions, obtained by rotating the X and Y axes by 45°. The $U_{Y'}$ component shown in Fig. 3b is now anti-symmetrical with respect to the [11] direction. As a consequence, a plane wave normally incident upon the ΓM direction cannot contribute to the transmission because this wave is symmetrical with respect to the Y' axis. As a conclusion, the dispersion curve labelled 1st in Fig. 2 contributes to a NPB to the transmission along ΓX direction but not to the transmission along ΓM. With similar calculations whose details are skipped, we have shown that the branch labelled 2nd contributes to the transmission spectrum along ΓM but not along ΓX.

The above class of phononic crystal with a NPB inside a wide frequency gap offers a good possibility of designing acoustic waveguides for selective frequency transmission. For instance, we have studied the transmission spectrum through a waveguide constituted by a row of hollow cylinders inside a phononic crystal composed of filled steel cylinders in water. Within the frequency gap of the phononic crystal, the waveguide can transmit at a given selected frequency. For example, when the inner radii of the hollow cylinders are r_i = 1.2 mm (respectively r_i = 1.0 mm) the transmission spectrum exhibits a narrow peak at 161 kHz (respectively 194 kHz). It is worthwhile to notice that these frequencies are significantly different from those obtained when the transmission occurs through a perfect phononic crystal composed of hollow cylinders. In the latter case, the frequencies of the NPB are respectively 155 and 184 kHz. This difference can be attributed to the confinement effect inside the waveguide which appears to be far from being negligible.

In Fig. 4, we show the transmission spectrum through a waveguide containing alternate hollow cylinders of inner radii equal to 1.2 and 1.0 mm respectively. One can notice that the transmission spectrum exhibits now two distinct peaks at 161 and 194 kHz, which makes this structure a good candidate for multiplexing purposes. In a recent work [5] we have also shown that such a waveguide transporting two selected frequencies can be divided into two channels (Y-shaped waveguide). Then, each frequency will be transmitted into only one channel, which means that such structure behaves as a demultiplexing device.

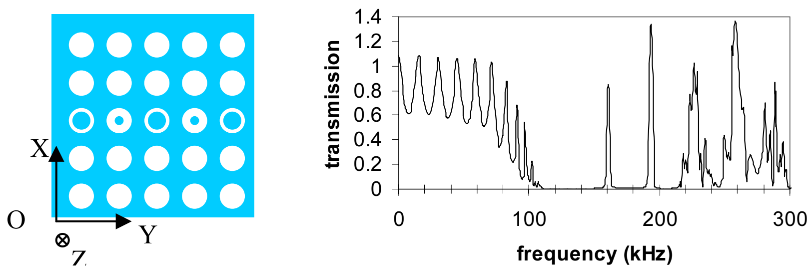


Fig. 4 (a) 2D cross section of NPB waveguide composed of hollow cylinders with large (1.2 mm) and small (1.0 mm) inner radii in alternation. (b) transmission spectrum of the mixed waveguide.

In conclusion, we have calculated both the transmission spectra and dispersion curves in phononic crystals constituted by hollow cylinders and explained the apparent differences between their behaviours. This class of phononic crystal can exhibits a NPB inside a wide frequency gap in their transmission spectrum. This opens the possibility of applications such as selective waveguiding, filtering and multiplexing phenomena when such hollow cylinders are inserted as a row of defect in a perfect phononic crystal.

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