Example 5.7

The results of three CD tests on a soil at failure are as follows:

Test number	σ'_3(kPa)	Deviatoric stress (kPa)
1	100	247.8 (peak)
2	180	362 (peak)
3	300	564 (no peak observed)

The detailed results for Test 1 are as follows. The negative sign indicates expansion.

Δz (mm)	ΔV (cm^3)	Plunger Load - P_z (N)
0	0.00	0.0
0.152	0.02	61.1
0.228	0.03	94.3
0.38	-0.09	124.0
0.76	-0.50	201.5
1.52	-1.29	257.5
2.28	-1.98	292.9
2.66	-2.24	298.9
3.04	-2.41	298.0
3.8	-2.55	279.2
4.56	-2.59	268.4
5.32	-2.67	252.5
6.08	-2.62	238.0
6.84	-2.64	229.5
7.6	-2.66	223.2
8.36	-2.63	224.3

The initial size of the sample is 38 mm in diameter and 76 mm in length.

(a) Determine the friction angle for each test.

(b) Determine τ_p, τ_{cs}, E', and E'_s at peak shear stress for Test 1.

(c) Determine ϕ'_{cs}.

(d) Determine α_p for Test 1.

Strategy

From a plot of deviatoric stress versus axial strain for Test 1, you will get τ_p, τ_{cs}, E' and E'_s. The friction angles can be calculated or found from Mohr's circle.

Solution 5.7

Step 1: Determine the friction angles.

Use a table to do the calculations.

Test No.	**σ'_3 kPa**	**σ'_1 - σ'_3 kPa**	**σ'_1 kPa**	**σ'_1 + σ'_3**	$\phi'_p = \sin^{-1}\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3}$
Test 1	100	247.8	347.8	447.8	33.6°
Test 2	180	362	542	722	30.1°
Test 3	300	564	864	1164	29°

Alternatively, plot Mohr's circles and determine the friction angles as shown for Test 1 and Test 2 in Fig. Ex. 5.7a.

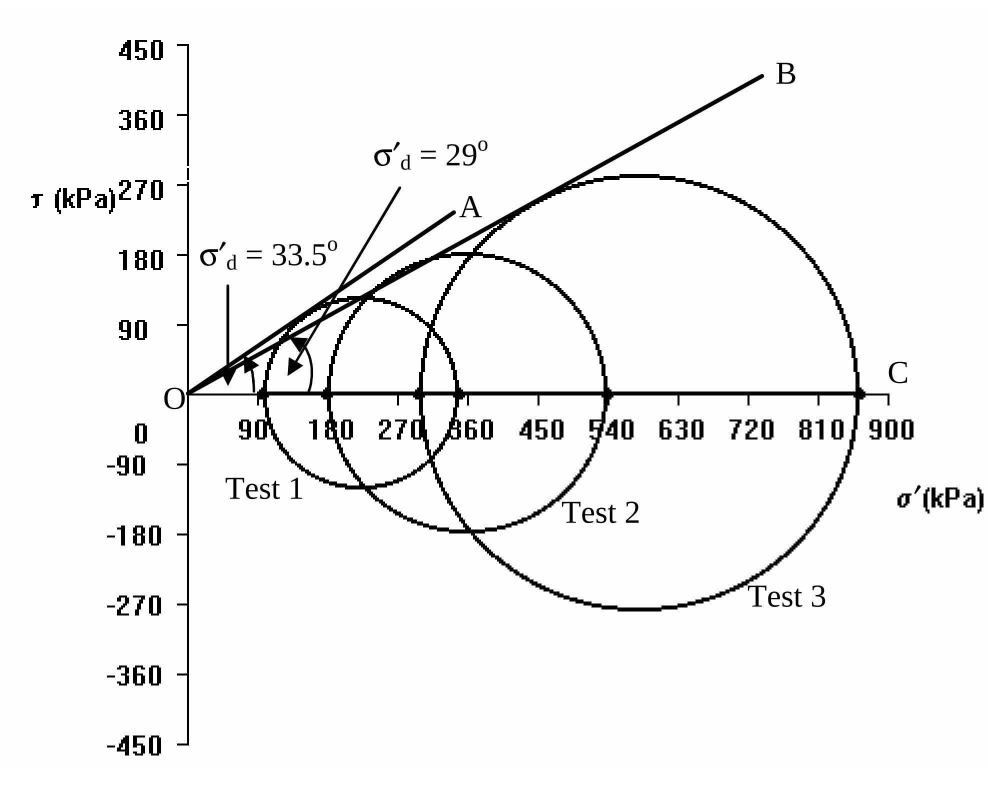

Fig. Ex. 5.7a

Step 3: Determine τ_p and τ_{cs} from a plot of deviatoric stress versus axial strain response for Test 1.

The initial area is:

$$A_o = \frac{\pi D_o^2}{4} = \frac{\pi \times 38^2}{4} = 1134 \text{ mm}^2$$

$$V_o = \frac{\pi d^2 H_o}{4} = \frac{\pi \times 38^2 \times 76}{4} = 86193 \text{ mm}^3$$

See table below for calculations and Fig. E.5.7b for a plot of the results.

A_o =	1134	mm^2			
Δz (mm)	ε_1	ΔV (cm^3)	ε_p	A	q = P_z/A
	Δz/h		$\Delta V/V_o$)	mm^2	kPa
0.00	0.00	0.00	0.00	1134	0.0
0.15	0.20	0.02	0.02	1136	53.8
0.23	0.30	0.03	0.03	1137	82.9
0.38	0.50	-0.09	-0.10	1141	108.7
0.76	1.00	-0.50	-0.58	1152	174.9
1.52	2.00	-1.29	-1.50	1175	219.2
2.28	3.00	-1.98	-2.30	1196	244.9
2.66	3.50	-2.24	-2.60	1206	247.8
3.04	4.00	-2.41	-2.80	1215	245.3
3.80	5.00	-2.55	-2.97	1229	227.1
4.56	6.00	-2.59	-3.01	1243	215.9
5.32	7.00	-2.67	-3.10	1257	200.8
6.08	8.00	-2.62	-3.05	1270	187.3
6.84	9.00	-2.64	-3.07	1285	178.7
7.60	10.00	-2.66	-3.09	1299	171.8
8.36	11.00	-2.63	-3.06	1313	170.7

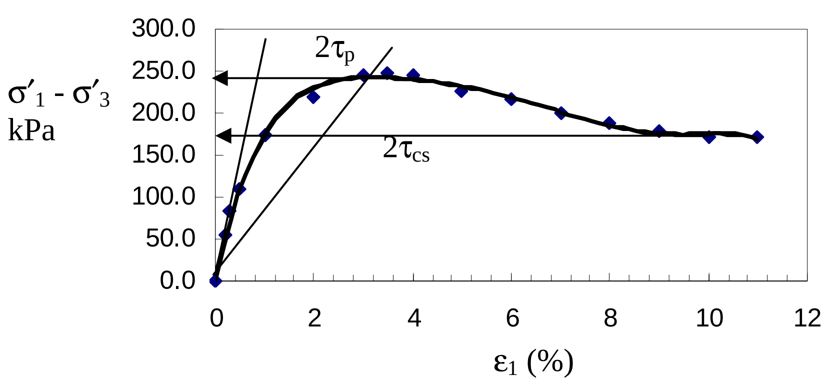

Fig. E5.7b

Extract τ_p and τ_{cs}

$$\tau_p = \frac{(\sigma_1' - \sigma_3')_d}{2} = \frac{247.8}{2} = 124\text{kPa},\ \tau_{cs} = \frac{(\sigma_1' - \sigma_3')_{cs}}{2} = \frac{170.7}{2} = 85.4\text{kPa}$$

Step 4: Determine E′and E_s

$$E' = \frac{53.8}{0.002} = 26{,}887\text{kPa}$$

$$E'_s = \frac{247.8}{0.035} = 7081\text{kPa}$$

Step 5: Determine ϕ'_{cs} $2\tau_p$

The deviatoric stress and the volumetric change appear to be constant from about $\varepsilon_1 \approx$ 10%. We can use the result at ε_1 = 11% to determine ϕ'_{cs}.

$(\sigma'_3)_{cs}$ = 100 kPa, $(\sigma'_1)_{cs}$ = 170.7 + 100 = 270.7kPa

$$\phi'_{cs} = \sin^{-1}\left(\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}\right)_{cs} = \sin^{-1} \frac{170.7}{270.7 + 100} = 27.4^o$$

Step 6: Determine α_p

$\alpha_p = \phi'_p - \phi'_{cs} = 33.6 - 27.4 = 6.2^o$